Kyoto University,
Graduate School of Economics
Research Project Center Discussion Paper Series

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Discussion Paper No. E-12-003

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May 2012

The Macroeconomic Effects of the Wage Gap between Regular and Non-Regular Employment and Minimum Wages*

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Abstract

We analyze the effect of the wage gap between regular and non-regular employment on a macroeconomy by using a Keynesian dynamic model. If the steady state equilibrium exhibits the stagnationist regime, the size of the reserve army effect affects the stability of the equilibrium. On the other hand, if the steady state equilibrium exhibits the exhilarationist regime, an increase in the wage gap destabilizes the equilibrium. The introduction of the minimum wage is desirable in that it mitigates fluctuations of business cycles. However, the introduction of an inappropriate minimum wage policy leads to the real wage and employment rate that are lower than the steady state values.

Keywords: wage gap; regular and non-regular employment; minimum wage; demand-led growth model

JEL Classification: E12; E24; E25; E32; J31; J83

*We would like to thank Peter Skott, Naoki Yoshihara, Takashi Ohno, Shinya Fujita, and seminar participants at Kyoto University, Nihon University, and Hitotsubashi University for their useful comments and suggestions.

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1 Introduction

Quite a long time has passed since the debate on the so-called regular and non-regular employment has begun in Japan. It has become an increasingly urgent problem.[1] We can find, as reasons for the aggravation of this situation, not only the increase of non-regular employment due to a prolonged economic depression but also the prevalence of a discriminative treatment based on the employment pattern, that is, regular employment and non-regular employment.[2] The progress of such an undesirable situation leads us to pay attention to the questions: how the expansion of the wage gap between regular and non-regular employees has an influence on a macroeconomy; and what a consequence of a greater increase in non-regular employees is. In order to investigate the questions, we build a model in which regular and non-regular workers coexist. Further, we consider the minimum wage policy as one of the policies taken by a government to alleviate the situation. We analyze how the introduction of the minimum wage policy affects the economy.

Flaschel and Greiner (2009) have justified a minimum wage policy based on human rights.[3] Flaschel and Greiner's model is an application of Goodwin's (1967) growth cycle model, which assumes the two-class economy that consists of working and capitalist classes. They show the existence of an endogenous and perpetual business cycle with respect to the employment rate and wage share. They also show that the introduction of the minimum wage within certain bounds below the steady state wage share can increase the stability of the economy. In their model, the minimum wage means the minimum wage share exogenously determined by the government because labor productivity is constant. Based on Flaschel and Greiner (2009), a line of research has been developed. Flaschel and Greiner (2011) and Flaschel et al. (2011) introduce into the Goodwin model heterogeneous labor, that is, skilled and unskilled workers. The labor market consists of the first and second markets: the former is the market only for the skilled workers and the latter is one for both the unskilled workers and the skilled workers who do not find a job in the first market. It is assumed that the skilled workers do not lose their job because they can always find a job in the second labor market even if they do not find a job in the first one. Thus, the excess supply of labor in the first labor market means that some of the unskilled workers lose their jobs with the

[1]For instance, according to the labor force survey February 2012 by the Ministry of Internal Affairs and Communications (http://www.stat.go.jp/data/roudou/longtime/03roudou.htm), the ratio of non-regular employee to the whole employee excluding executive of company or corporation of 2011 October-December average is 35.7 %, the highest record ever registered. Especially, that of the younger generation from 15 to 34 is 33.1%.

[2]Japan does not ratify the International Labor Organization (ILO) convention 111, which prescribes for a prohibition of discriminating in respect of employment and occupation, in particular, a discrimination of regular and non-regular employee.

[3]As a theoretical contribution to researches on human rights, see, for example, Sen (2004).

assumption that labor supply is abundant. In addition, it is also assumed that only the skilled workers in the first labor market are involved in participating in the wage bargain. This means that wages of the workers in the second labor market depend on the bargain by the skilled workers. Thus, the skilled workers have a large influence on the unskilled workers in the second labor market with respect to their wage determination and employment. Then, they show, like Flaschel and Greiner (2009), that endogenous business cycles can occur, and that the introduction of the minimum wage in the second labor market can alleviate fluctuations of the economy.

The model developed in the present paper can be seen as a line of the above contributions. We consider a two-class economy, which is the same as Flaschel and Greiner (2009). In addition, two kinds of heterogeneous labor, that is, regular and non-regular employments are introduced within the working class. Thus, our model is the two-class economy that consists of three units, which is similar to Flaschel and Greiner (2011). We analyze the effect of the wage gap between regular and non-regular employments on the stability of the steady state equilibrium by using a Kaleckian model. We also analyze how business cycles are affected by the introduction of the minimum wage.

The so-called Kaleckian model is a kind of Keynesian dynamic models based on the principle of the effective demand, whose basic framework was developed by Rowthorn (1981). In his model, two kinds of labor, that is, direct labor and indirect labor, which are emphasized by Kalecki himself, are considered. Direct labor refers to variable labor in accordance with a change in output, while indirect labor refers to fixed labor regardless of the change of output. In this sense, we can interpret direct (indirect) labor as regular (non-regular) one. Although Rowthorn (1981) emphasized two kinds of labor, surprisingly, almost all Kaleckian models do not consider it.[4] In particular, only a few contributions analyze the effect of direct and indirect labor on the stability of the steady state equilibrium. Raghavendra (2006), one of the exceptional cases, extends Rowthorn's model and presents a model in which an endogenous business cycle with respect to the two variables (the capacity utilization rate and profit share) occurs. To be concrete, although the income distribution (the wage and profit shares) is exogenously given in Rowthorn's model, it is endogenously determined in Raghavendra's one. In place of the investment function in Rowthorn's model, which is an increasing function of the capacity utilization and the profit rate, he introduces an investment function presented by Marglin and Bhaduri (1990), which is an increasing function of the capacity utilization and the profit share. Moreover, he adopts a non-linear investment function.

[4]For the literature considering the two kinds of labor in a Kaleckian model, see Lavoie (1992). Lavoie (2009) is a recent contribution to the field.

In Raghavendra's model, a limit cycle occurs under certain conditions due to the interaction between the nonlinearity of the investment function and increasing returns to scale caused by the existence of fixed labor. That is to say, the occurrence of an endogenous and perpetual business cycle is proved by Poincaré-Bendixson theorem. This suggests an alternation of the exhilarationist and stagnationist regimes, where the exhilarationist (stagnationist) regime means that an increase in the profit share increases (decreases) the capacity utilization rate.[5] In this paper, fixed and variable labor in Raghavendra model are interpreted as regular and non-regular employments, respectively. Furthermore, we present the extended model in some respects, and analyze the stability of the steady state equilibrium and the effect of an introduction of minimum wages on a business cycle.

There are two differences between our model and Raghavendra one. First, an assumption concerning wage differs. Whereas Raghavendra (2006) assumes an equal pay between fixed labor and variable one, we assume that regular workers earn a higher wage than non-regular workers. Second, a formulation regarding dynamics of income distribution differs. In his model, the dynamics of income distribution (the profit share) are derived from the dynamic markup pricing rule of firms and the wage curve.[6] In contrast, we adopt the conflicting-claims theory of inflation in order to obtain the dynamics of income distribution.[7] In particular, we introduce the reserve army effect that the target profit share of labor unions is a decreasing function of the capacity utilization rate.

We analyze the dynamics of the capacity utilization rate and profit share by using the extended model. In the case where the steady state equilibrium is unstable in the exhilarationist regime, under certain conditions, an endogenous and perpetual business cycle occurs by the Hopf bifurcation theorem. Generally, the longer a downturn of a business cycle is, the worse a non-regular worker becomes. One of the policies that a government should take to alleviate such an undesirable situation is a minimum wage policy, which puts lower bounds to the wage share, that is, upper bounds to the profit share.

Let us explain the minimum wage policy. Here, the minimum wage is introduced by setting the upper bound of the profit share. Such formalization can be regarded as the minimum wage in our model because the determination of the maximum profit share is equivalent to that of the minimum wage of the non-regular employment. To be concrete, the reason is as follows. First, by definition, the maximum profit share is equal to the minimum wage share. Second, given the labor productivity, the determination of the minimum wage share is equiv-

[5]See Blecker (2002) for further discussion concerning various regimes in the Kaleckian model.

[6]The wage curve in Raghavendra (2006) means that a level of wage is an increasing function of the capacity utilization rate.

[7]Flaschel and Greiner (2009) present an extended version of the Goodwin model, where the dynamics of income distribution are given by the way that the dynamic markup pricing rule and a rate of change in wage depends on both capacity utilization and inflation rate.

alent to that of the minimum average wage of the whole economy. Third, the average wage of the whole economy is proportional to the wage of the non-regular employment.[8] Thus, from the three facts, the equivalence between the determinations of the maximum profit share and the minimum wage of the non-regular employment holds.

In our analysis, we focus on the case where the steady state equilibrium is locally unstable. The government does not accurately know the equilibrium profit share. Thus, the minimum wage determined by the government does not necessarily coincide with it. In this case, the following results are obtained by our analysis.

Under the exhilarationist regime, setting a higher profit share, that is, a lower wage one relative to the equilibrium, diminishes the amplitude of the business cycle. In contrast, setting a lower profit share, that is, a higher wage one relative to the equilibrium, the economy converges to a point that is different from the equilibrium. In this case, we are faced with the problem with respect to the comparison between the steady state without minimum wage and the equilibrium after introducing it. We can compare the two equilibria by means of an iso-real wage curve. Each value of the two equilibria depends on the shape of the curve.

Under the stagnationist regime, for any minimum wage relative to the steady state profit share, the economy converges to the steady state equilibrium (as long as the minimum and maximum wages are simultaneously introduced). In this case, in accordance with a minimum wage being put above, below, or equivalent to the steady state profit share, the economy converges to the corresponding value. In comparing among the three equilibria, contrary to the above case, for any shape the iso-real wage curve has, we can completely order the three equilibria.

The rest of the paper is organized as follows. Section 2 provides our model. Section 3 analyzes the properties of the steady state equilibrium and its stability. Section 4, given tha the steady state equilibrium is locally unstable, analyzes how the introduction of the minimum wage policy affects the economy. Section 5 concludes.

2 The model

We consider an economy in which there exist two heterogeneous workers (regular and non-regular employments) and capitalists. Workers' income consists of wages only, and they spend all their income for consumption. On the other hand, capitalists obtain their income from profits only, which is saved at a constant rate s. Then, the saving function is given by

$$g_s = sr, \quad 0 < s < 1, \tag{1}$$

[8] See equations (7) and (8) below.

where $g_s = S/K$ is the ratio of real savings S to the capital stock K, and r is the profit rate.

Following Marglin and Bhaduri (1990), we assume that the investment function of firms is an increasing function of the capacity utilization rate u and the profit share m.[9]

$$g_d = g_d(u, m), \quad g_{du} > 0, \; g_{dm} > 0, \tag{2}$$

where $g_d = I/K$ denotes the ratio of the real investment I to the capital stock, g_{du} denotes the partial derivative of the investment function with respect to the capacity utilization, and g_{dm} denotes the partial derivative of the investment function with respect to the profit share. For simplicity, we do not consider capital depreciation.

Let us assume that the ratio of the potential output Y^F to the capital stock is technically fixed, that is, always constant. Then, the capacity utilization rate can be represented as $u = Y/K$, where Y denotes the actual output. Note that the relation $r = mu$ holds among the profit rate, the profit share, and the capacity utilization rate. In the following analysis, without loss of generality, we assume that the ratio of the potential output to the capital stock is equal to unity.

We assume that the regular employment L_r is related to the potential output while the non-regular employment L_{nr} is related to the actual output.

$$L_r = \alpha Y^F, \quad \alpha > 0, \tag{3}$$

$$L_{nr} = \beta Y, \quad \beta > 0, \tag{4}$$

where α and β are positive constants. The ratio of the regular employment to the non-regular employment leads to $L_r/L_{nr} = \alpha/(\beta u)$. This means that an increase in the capacity utilization rate leads to a decrease in this ratio. In other words, relatively many non-regular workers are employed with a rise in the capacity utilization rate. Once the capacity utilization rate is determined at the steady state equilibrium, the ratio of the regular employment to the non-regular employment is also determined.

We assume that a quantity adjustment that the capacity utilization rate increases (decreases) in accordance with an excess demand (supply) in the goods market.

$$\dot{u} = \phi(g_d - g_s), \quad \phi > 0, \tag{5}$$

where the parameter ϕ denotes the speed of adjustment of the goods market.

We define the level of the average labor productivity of the economy as $a = Y/L$, where L

[9] In the typical Kaleckian model, the investment function is formalized as an increasing function of the profit rate and the capacity utilization rate. On the contrary, Marglin and Bhaduri (1990) assert that one of the variables of the investment function should be the profit share rather than the profit rate.

denotes the aggregate employment, that is, $L = L_r + L_{nr}$. From this, we obtain the following:

$$a = \frac{Y}{\alpha Y^F + \beta Y} = \frac{u}{\alpha + \beta u}. \tag{6}$$

This implies that the average labor productivity is an increasing function of the capacity utilization rate, that is, increasing returns to scale prevail. Since u is constant at the steady state equilibrium, the corresponding average labor productivity is also constant. Thus, there is no perpetual technical progress in our model.

The nominal wage of the regular employment w_r is supposed to be higher than that of the non-regular employment w_{nr} at a certain rate γ.[10]

$$w_r = \gamma w_{nr}, \quad \gamma > 1. \tag{7}$$

From these, the average wage of the economy leads to

$$w = \frac{L_r}{L} w_r + \frac{L_{nr}}{L} w_{nr} = \frac{\alpha}{\alpha + \beta u} w_r + \frac{\beta u}{\alpha + \beta u} w_{nr} = \left[\frac{\gamma\alpha + \beta u}{\gamma(\alpha + \beta u)} \right] w_r. \tag{8}$$

The average wage is given by the weighted average between the wages of regular and non-regular employments. Each weight corresponds to the corresponding employment share. Whereas the weight of the regular employment is a decreasing function of the capacity utilization rate, the weight of the non-regular employment is an increasing function of the capacity utilization. The component in the square bracket is a decreasing function of the capacity utilization rate.

Next, we formalize the equation of the price of goods p and the equation of the wage of the regular employment by using the theory of conflicting-claims inflation.[11] First, firms set their price so as to narrow the gap between the target profit share of firms m_f and the actual profit share, and accordingly, the price changes. Second, labor unions negotiate so as to narrow the gap between the target profit share of labor unions m_w and the actual profit share, and accordingly, the nominal wage of the regular employment changes. The two assumptions can be written as follows:

$$\frac{\dot{p}}{p} = \theta(m_f - m), \quad 0 < \theta < 1, \ 0 < m_f < 1, \tag{9}$$

$$\frac{\dot{w}_r}{w_r} = (1 - \theta)(m - m_w), \quad 0 < m_w < 1, \tag{10}$$

[10]A similar formalization is adopted by Lavoie (2009).

[11]The theory of conflicting-claims inflation is presented by Rowthorn (1977). For previous studies on Kaleckian models with this theory, see, for example, Dutt (1987) and Cassetti (2003).

where θ is a positive parameter. We interpret θ and $1-\theta$ as the bargaining power of firms and that of labor unions, respectively. Further, by taking the reserve army effect into consideration, we assume that m_w is a decreasing function of the capacity utilization rate.

$$m_w = m_w(u), \quad m_w' < 0. \tag{11}$$

Since the profit share is given by $m = 1-(wL/pY)$ by definition, taking the derivative of m with respect to time, we obtain

$$\frac{\dot{m}}{1-m} = \frac{\dot{p}}{p} - \frac{\dot{w}}{w} + \frac{\dot{a}}{a}. \tag{12}$$

From equations (8) and (10), the rate of change of the average wage in the whole economy is given by

$$\frac{\dot{w}}{w} = -\frac{(\gamma-1)\alpha\beta}{(\gamma\alpha+\beta u)(\alpha+\beta u)}\cdot\dot{u} + (1-\theta)[m-m_w(u)]. \tag{13}$$

From equation (6), the rate of change of the labor productivity is given by

$$\frac{\dot{a}}{a} = \frac{\alpha}{(\alpha+\beta u)u}\cdot\dot{u}. \tag{14}$$

By substituting equations (1) and (2) in equation (5), and also equations (9), (12), and (13) in (11), we can obtain the following dynamic equations with respect to the capacity utilization rate and the profit share.

$$\dot{u} = \phi[g_d(u,m) - smu], \quad \phi > 0, \tag{15}$$

$$\dot{m} = -(1-m)[m-\theta m_f - (1-\theta)m_w(u) - f(u)\dot{u}], \quad f(u) = \frac{\alpha\gamma}{(\gamma\alpha+\beta u)u}, \; f'(u) < 0. \tag{16}$$

Firms set m_f as large as possible. Conversely, labor unions set m_w as small as possible. Hence, we can assume that $m_f > m_w(u)$.

3 Characteristics of the steady state equilibrium and its stability

3.1 Characteristics of the steady state equilibrium

The steady state equilibrium is an equilibrium such that $\dot{u} = \dot{m} = 0$. From this, we have the simultaneous equation with respect to u^* and m^*.

$$g_d(u^*, m^*) = sm^*u^*, \tag{17}$$

$$m^* = \theta m_f + (1 - \theta)m_w(u^*). \tag{18}$$

In the following analysis, we assume that there exists a unique pair of $u^* \in (0, 1)$ and $m^* \in (0, 1)$ that simultaneously satisfies equations (17) and (18).

The capacity utilization rate and the profit share at the steady state equilibrium depend on the bargaining power, the target profit share of firms, and the target profit share of labor unions. However, the steady state equilibrium does not depend on the four parameters, ϕ, γ, α and β. This property will be used in the analysis of the next section.

3.2 The stability of the steady state equilibrium

In order to investigate the stability of the steady state equilibrium, we analyze the Jacobian matrix of the system of the differential equations. Let the Jacobian matrix be $\mathbf{J}$. The matrix $\mathbf{J}$ is a 2×2 matrix, and its components are given as follows:

$$J_{11} = \frac{\partial \dot{u}}{\partial u} = \phi(g^*_{du} - sm^*), \tag{19}$$

$$J_{12} = \frac{\partial \dot{u}}{\partial m} = \phi(g^*_{dm} - su^*), \tag{20}$$

$$J_{21} = \frac{\partial \dot{m}}{\partial u} = (1 - m^*)[(1 - \theta)m'_w(u^*) + f(u^*)J_{11}], \tag{21}$$

$$J_{22} = \frac{\partial \dot{m}}{\partial m} = -(1 - m^*)[1 - f(u^*)J_{12}]. \tag{22}$$

Each component of $\mathbf{J}$ is evaluated at the steady state equilibrium.

Let us assume the following condition:

Assumption 1. $sm^* > g^*_{du}$.

This means that the response of savings to the capacity utilization rate is larger than that of investments. This assumption makes the quantity adjustment of the goods market stable.

Assumption 1 is sometimes called the Keynesian stability condition (Marglin and Bhaduri, 1990), which is often imposed in Kaleckian models. With Assumption 1, we can obtain $J_{11} < 0$.

Let us classify the regime according to the effect of an increase in the profit share on the capacity utilization.

Definition 1. *If the relation $g^*_{dm} < su^*$ holds, the steady state equilibrium is called the stagnationist regime. If, on the other hand, the relation $g^*_{dm} > su^*$ holds, the steady state equilibrium is called the exhilarationist regime.*[12]

If the response of investment to the profit share is less than that of saving, then the steady state equilibrium exhibits the stagnationist regime. On the other hand, if the response of the investment to the profit share is more than that of saving, then the steady state equilibrium exhibits the exhilarationist regime. Depending on which regime is realized in the steady state equilibrium, the stagnationist regime or the exhilarationist regime, we have $J_{12} < 0$ or $J_{12} > 0$.

The steady state equilibrium is locally stable if and only if the determinant of the Jacobian matrix $\mathbf{J}$ is positive and the trace is negative. Let us confirm this fact in our model.

First, calculating the determinant, we have

$$\det \mathbf{J} = -(1 - m^*)[J_{11} + (1 - \theta)m'_w(u^*)J_{12}]. \tag{23}$$

When the steady state equilibrium exhibits the exhilarationist regime, that is, $J_{12} > 0$, $\det \mathbf{J} > 0$ always holds. On the contrary, when the steady state equilibrium exhibits the stagnationist regime, that is, $J_{12} < 0$, $\det \mathbf{J} < 0$ holds if the absolute value of $m'_w(u)$ is sufficiently large. Further, when $\det \mathbf{J} < 0$, the discriminant of the characteristic equation is always positive. Hence, we obtain the two distinct real roots.

Second, calculating the trace, we have

$$\operatorname{tr} \mathbf{J} = J_{11} - (1 - m^*) + (1 - m^*)f(u^*)J_{12}. \tag{24}$$

By assumption, $J_{11} < 0$. However, since J_{12} can be positive or negative, $\operatorname{tr} \mathbf{J}$ is not always negative. When the steady state equilibrium exhibits the stagnationist regime, that is, $J_{12} < 0$, $\operatorname{tr} \mathbf{J} < 0$ always holds. On the contrary, when the steady state equilibrium exhibits the exhilarationist regime, that is, $J_{12} > 0$, $\operatorname{tr} \mathbf{J} > 0$ can hold.

[12] The exhilarationist regime is generally defined as a regime such that an increase in the exogenously given profit share leads to a rise in the capacity utilization rate at the steady state equilibrium. However, we cannot apply this definition to our model because the profit share at the steady state equilibrium is an endogenous variable. Here, we follow the definition of Raghavendra (2006).

From the above analysis, we obtain the following propositions.

Proposition 1. *Suppose that the steady state equilibrium exhibits the stagnationist regime. If the reserve army effect is small, then the steady state equilibrium is locally stable. On the other hand, if the reserve army effect is large, then the steady state equilibrium is locally unstable.*

Let us explain Proposition 1 intuitively. This proposition means that, in the stagnationist regime, the higher reserve army effect has unstable effects on the economy. Consider the case where the capacity utilization rate deviates from the steady state capacity utilization rate due to the occurrence of an exogenous shock. Suppose that the capacity utilization rate is less than the steady state equilibrium, for instance. In this case, there exist two opposing effects. First, the capacity utilization rate increases due to $J_{11} < 0$, which is called the direct stable effect. Second, the fall in the capacity utilization rate leads to an increase in the profit share due to $J_{21} < 0$. As a consequence, due to $J_{12} < 0$, the capacity utilization rate decreases, which is called the indirect unstable effect. Thus, in the stagnationist regime, whereas the direct effect is stable, the indirect effect is unstable. If the reserve army mechanism effect is large, the profit share fairly decreases due to a strong effect of $J_{12} < 0$. As a result, the capacity utilization rate largely increases via an effect of J_{12}. That is, the indirect unstable effect works intensely. In the unstable case, the steady state equilibrium is a saddle point.

Proposition 2. *Suppose that the steady state equilibrium exhibits the exhilarationist regime. Then, the steady state equilibrium can be locally unstable.*

This proposition means that, in the exhilarationist regime, the higher response of the capacity utilization rate to a change in the profit share can destabilize the economy. First, in the exhilarationist regime, if the capacity utilization rate is less than the steady state value, then it increases due to $J_{11} < 0$. This is the direct effect. Second, if the capacity utilization rate is less than the steady state value, then the profit share increases $J_{21} < 0$, which increases the capacity utilization rate due to $J_{12} > 0$. This is the indirect effect. In the exhilarationist regime, both the direct and indirect effects seem stable. However, if the indirect effect is too large, that is, if the response of the capacity utilization rate to the profit share is sufficiently large, the capacity utilization rate fairly increases due to the strong effect of $J_{12} > 0$. In this case, the capacity utilization rate increases too much and deviates from the steady state, and consequently, the steady state equilibrium will be unstable.

Now, we show that endogenous and perpetual business cycles occur. By rearranging $\mathrm{tr}\,\mathbf{J}$, we obtain:

$$\operatorname{tr}\mathbf{J} = \phi A - (1 - m^*), \quad A \equiv (g^*_{du} - sm^*) + (1 - m^*)f(u^*)(g^*_{dm} - su^*). \tag{25}$$

The term A does not depend on the parameter ϕ that represents the speed of adjustment of the goods market. Both the capacity utilization rate and the profit share at the steady state equilibrium do not also depend on ϕ. Thus, we can choose ϕ as a bifurcation parameter.

Proposition 3. *Suppose that the steady state exhibits the exhilarationist regime and that the term A is positive. Then, a limit cycle occurs when the speed of adjustment of the goods market lies within some range.*

Proof. Put $\phi_0 = (1 - m^*)/A > 0$. Taking a positive value ϕ arbitrarily, we have $\operatorname{tr}\mathbf{J} = 0$ for $\phi = \phi_0$, $\operatorname{tr}\mathbf{J} < 0$ for $\phi < \phi_0$, and $\operatorname{tr}\mathbf{J} > 0$ for $\phi > \phi_0$. Thus, $\phi = \phi_0$ is a Hopf bifurcation point. That is, there exists a continuous family of non-stationary, periodic solutions of the system around $\phi = \phi_0$. ■

This proposition means that, as the speed of adjustment of the goods market becomes faster, there appears a point at which the stable steady state equilibrium switches to the unstable one. As Raghavendra (2006) has shown, we also have a similar result: with regard to the phase such that both the profit share and the capacity utilization rate simultaneously increase (decrease), and the phase such that one increases while the other decreases, the former phase alternates with the latter. This means an apparent alternation of the exhilarationist and stagnationist regimes.

Next, we consider the parameter γ that represents the wage gap between regular and non-regular employments. Note that γ does not affect the equilibrium. Therefore, γ appears only in the part $f(u^*)$ of $\operatorname{tr}\mathbf{J}$. Furthermore, we find that

$$\frac{\partial f(u;\gamma)}{\partial \gamma} = \frac{\alpha\beta}{u(\gamma\alpha + \beta u)^2} > 0. \tag{26}$$

This means that, in the case where the steady state equilibrium exhibits the exhilarationist regime, a rise in γ amplifies the instability of the steady state equilibrium.

Proposition 4. *Suppose that the steady state equilibrium exhibits the exhilarationist regime. Then, an expansion of the gap between regular and non-regular employments makes the steady state equilibrium unstable.*

Furthermore, we obtain the following proposition.

Proposition 5. *Suppose that the steady state equilibrium is unstable in the exhilarationist regime. Then, a limit cycle occurs within a certain range of the wage gap between regular and non-regular employments.*

Proof. Put

$$\gamma_0 = \frac{\beta}{\alpha} \cdot \frac{(u^*)^2[(1-m^*) - J_{11}]}{\{[J_{11} - (1-m^*)]u^* + (1-m^*)J_{12}\}} > 0.$$

Taking a positive value γ arbitrarily, we have $\mathrm{tr}\,\mathbf{J} = 0$ for $\gamma = \gamma_0$, $\mathrm{tr}\,\mathbf{J} < 0$ for $\gamma < \gamma_0$, and $\mathrm{tr}\,\mathbf{J} > 0$ for $\gamma > \gamma_0$. Thus, $\gamma = \gamma_0$ is a Hopf bifurcation point. That is, there exists a continuous family of non-stationary, periodic solutions of the system around $\gamma = \gamma_0$. ■

As the wage gap expands, there appears a point at which the stable steady state equilibrium switches to the unstable one. Thus, an increase in the wage gap makes the economy more unstable. In order to stabilize the economy, it is desirable to shrink the wage gap between regular and non-regular employments.

Note that the steady state capacity utilization rate does not depend on parameters α and β. Given the steady state capacity utilization rate, we consider the effect of a rise in β/α on the stability of the steady state equilibrium. Then, a rise in β/α means that relatively many non-regular employees are employed compared to regular employees. The ratio β/α appears only in the part $f(u^*)$ of $\mathrm{tr}\,\mathbf{J}$. From equation (16), we find that

$$\frac{\partial f(u;\beta/\alpha)}{\partial(\beta/\alpha)} = -\frac{u^2}{u[\gamma + (\beta/\alpha)u]^2} < 0. \tag{27}$$

This means that, in the case where the the steady state equilibrium exhibits the exhilarationist regime, the higher the ratio β/α is, the smaller the $\mathrm{tr}\,\mathbf{J}$ is. From this, we obtain the following proposition.

Proposition 6. *Suppose that the steady state equilibrium exhibits the exhilarationist regime. Then, a rise in relatively many non-regular employees compared with regular employees makes the steady state equilibrium stable.*

This proposition shows a counter-intuitive result that an increase in relatively many non-regular employees will have an effect of making the economy unstable. Thus, if the government tries to decrease relatively many non-regular employees, the economy becomes unstable. However, we cannot conclude that a government should take an employment policy to increase non-regular employment in order to stabilize the economy. Rather, from propositions 4 and 6, we emphasize a policy mix between wage and employment policies;

whereas the economy becomes unstable due to a decrease in non-regular employees, a government makes an effort to stabilize the economy by conducting a policy to shrink the wage gap between regular and non-regular employments.

4 Introduction of minimum wages

As one of the policies taken by the government, we consider a minimum wage policy. The minimum wage is introduced by setting the upper bound of the profit share. We consider such formalization to be justifiable because the determination of the maximum profit share is equivalent to that of the minimum wage of the non-regular employment. The equivalence is obtained from the following three facts. First, by definition, the introduction of the maximum profit share ($m_{\max}$) is equal to that of the minimum wage share ($1 - m_{\max}$). Second, given the labor productivity ($a = Y/L$), the determination of the minimum wage share is equivalent to that of the minimum average wage of the whole economy, that is, $1 - m_{\max} = w/(pa)$. Third, by substituting equation (7) in (8), the average wage of the whole economy is proportional to the wage of the non-regular employment, that is, $w = \left(\frac{\gamma\alpha+\beta u}{\alpha+\beta u}\right) w_{nr}$, where $(\cdot) > 1$ for all $u \in (0, 1)$. Note that the minimum average wage of the economy $w_{\min}$ corresponding to $m_{\max}$ lies between w_{nr} and w_r from equation (8) and the third fact, that is, $w_r > w > w_{\min} > w_{nr}$.[13]

Here, in the same way as Flaschel and Greiner (2009, 2011) and Flaschel et al. (2011), we analyze how the introduction of the minimum wage affects the dynamics of the profit share and the capacity utilization rate.

In this section, in order to facilitate our analysis, numerical simulations will be used. For this purpose, we need to specify the investment function and the target profit share of labor unions. Here, we assume the following functional forms:

$$g_d = \psi u^{\delta} m^{\varepsilon}, \quad 0 < \delta < 1, \ \varepsilon > 0, \tag{28}$$

$$m_w = \sigma_0 - \sigma_1 u, \quad 0 < \sigma_0 < 1, \ \sigma_1 > 0. \tag{29}$$

If Okun's law, that is, a positive correlation between the employment rate and the capacity utilization rate, holds, we can regard the capacity utilization rate as a proxy variable of the employment rate. Such a method is adopted by Tavani et al. (2011).[14]

[13]To be concrete, the value of the bracket in equation (8) is smaller than unity for any u. This means that the relation $w \leq w_r$ holds. Equation (8) holds in the case where $u = 0$. Again, the value $\left(\frac{\gamma\alpha+\beta u}{\alpha+\beta u}\right)$ of the third fact is greater than unity for any u, which means that the relation $w > w_{nr}$ holds.

[14]For Kaleckian models that consider the endogenous determination of the employment rate, see Sasaki (2010, 2011, 2012a, 2012b)

4.1 The case of the exhilarationist regime

In this subsection, we assume that the steady state equilibrium exhibits the exhilarationist regime and that a limit cycle occurs.

We set the parameters as follows:

$$\psi = 0.8,\ \delta = 0.2,\ \varepsilon = 2,\ \phi = 8,\ s = 0.6,\ \theta = 0.4,$$
$$m_f = 0.7,\ \sigma_0 = 0.2,\ \sigma_1 = 0,\ \alpha = 1,\ \beta = 1.2,\ \gamma = 2,\ m_{\max} = 0.6.$$

Calculating the steady state with the above parameters, we obtain $u^* = 0.46$ and $m^* = 0.4$.

Given the arbitrary initial value, a limit cycle occurs (see Figure 1). The limit cycle is stable. That is, if the initial value is put far away from the steady state, it converges to the limit cycle. Even if the initial value is put at the neighborhood of the steady state, it converges to the limit cycle.

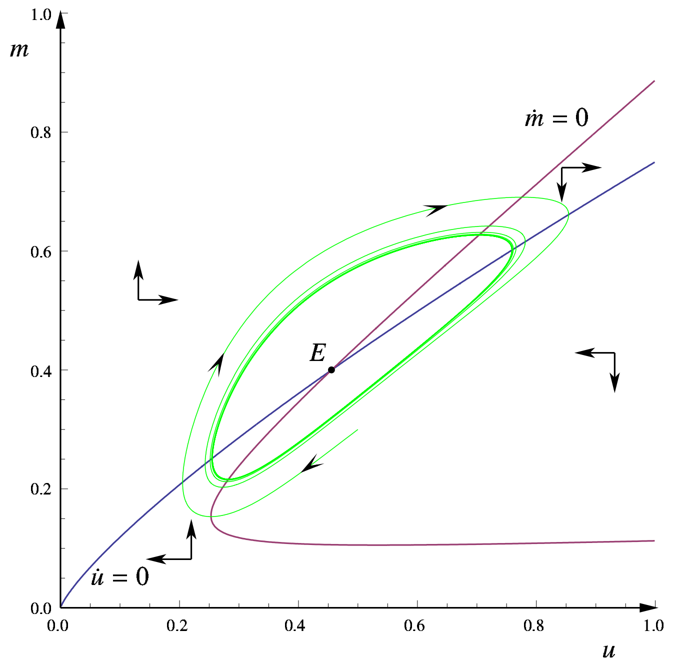

Figure 1: The occurrence of the limit cycle

Here, we introduce $m_{\max}$. Depending on how the government sets $m_{\max}$, we obtain different results.

First, we consider the case where exogenously given $m_{\max}$ is more than the maximum value (i.e., the top) of the limit cycle without introducing minimum wages (for instance, $m_{\max} = 0.8$). From the phase diagram, we can see that the initial value in this case converges to the limit cycle faster than the initial value in the case without $m_{\max}$.

Second, we consider the case where $m_{\max}$ is set between the maximum value of the limit cycle and the steady state equilibrium. Then, from the analysis of the phase diagram, we find that the size of the limit cycle becomes smaller. This means that the size of fluctuations decreases. In other words, by introducing the minimum wage, the size of business cycles can be reduced. Figure 2 shows the limit cycles in the case where $m_{\max} = 0.6$. Figure 3 shows the limit cycles of these two cases before and after introducing the minimum wage. Clearly, we can see that the size of business cycles diminishes.

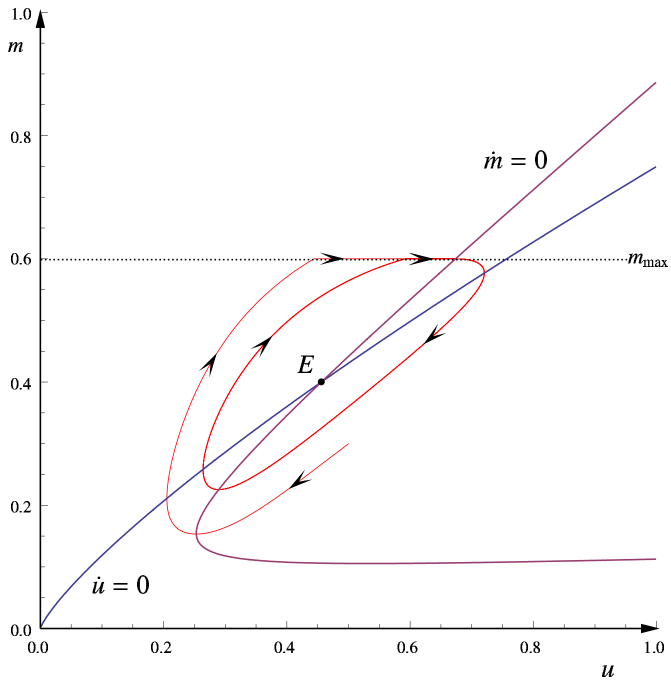

Figure 2: Introduction of the minimum wage

Third, we consider that $m_{\max}$ is exactly the same as the steady state profit share. Then, from the phase diagram, the economy converges to the steady state almost without fluctuations. That is, if the government knows the steady state equilibrium in advance, by putting the level of the minimum wage at the same level of the steady state, it can lead the economy to converge to the steady state.

Fourth, we consider the case where $m_{\max}$ is set less than the steady state equilibrium. If the government does not actually know the equilibrium, it is significant to consider such a case. Then, from the phase diagram, the economy converges to a point on line $\dot{u} = 0$ almost without fluctuations. That is, although fluctuations can be diminished, both the profit share and the capacity utilization rate obtained in the long run become smaller than those

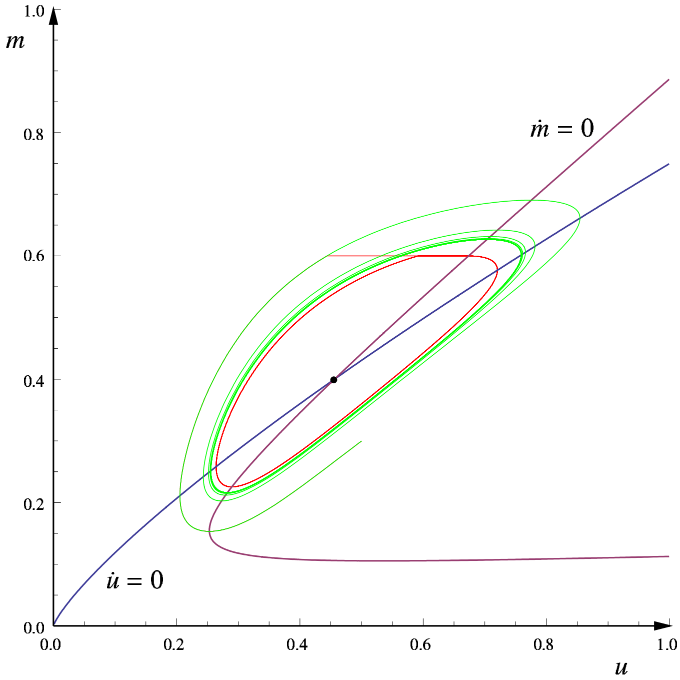

Figure 3: Comparisons of the two cycles

obtained before the introduction of the minimum wage policy. Figure 4 shows the case where $m_{\max} = 0.35$.

Let us compare the real wage of point E and that of point P. The real wage of regular employment and that of non-regular employment are given by using the capacity utilization rate and the profit share as follows:[15]

$$\frac{w_r}{p} = \frac{\gamma u}{\gamma\alpha + \beta u}(1 - m), \tag{30}$$

$$\frac{w_{nr}}{p} = \frac{u}{\gamma\alpha + \beta u}(1 - m). \tag{31}$$

From this, the real wage is an increasing function of the capacity utilization rate, and a decreasing function of the profit share. By rearranging the formulae, we obtain the iso-real wage curves in the (u, m)-space.

$$m = 1 - \frac{1}{\gamma}\frac{w_r}{p}\left(\frac{\alpha\gamma}{u} + \beta\right), \tag{32}$$

[15]Taking the derivatives of equations (30) and (31) with respect to γ, we obtain $\frac{\partial(w_r/p)}{\partial\gamma} > 0$ and $\frac{\partial(w_{nr}/p)}{\partial\gamma} < 0$, respectively. This means that an expansion of wage gap between regular and non-regular employments leads to an increase in real wage of regular employees and a decrease in real wage of non-regular employees.

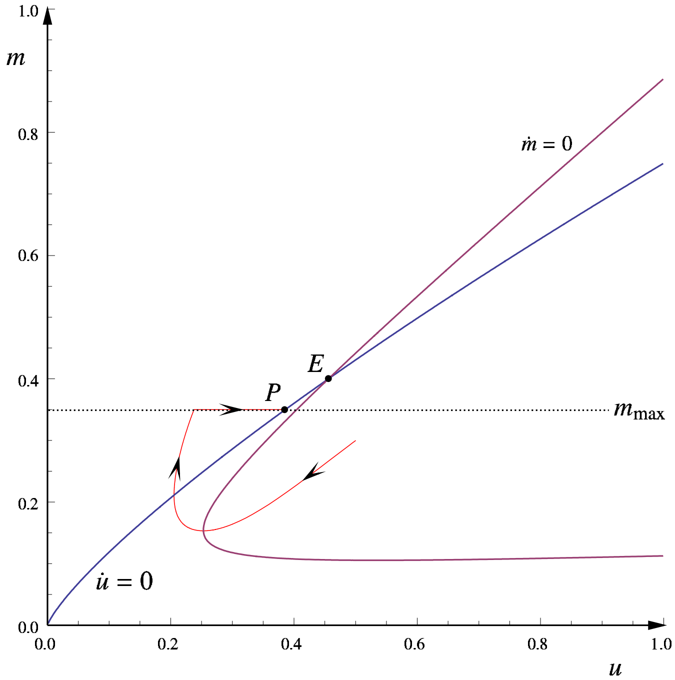

Figure 4: Introduction of the minimum wage that is larger than the equilibrium

$$m = 1 - \frac{w_{nr}}{p}\left(\frac{\alpha\gamma}{u} + \beta\right). \tag{33}$$

Each curve becomes a rectangular hyperbola. In the first quadrant of the (u, m)-space, the lower right-hand side these curves are located in, the higher real wages are associated with.

Which point E or P gives a higher real wage depends on two things: the shape of the iso-real wage curve and relative positions between points E and P. Iso-real wage curves and points E and P are shown in Figure 5. The real wage associated with the dotted iso-real wage curve is higher than that associated with the solid one. In the left figure, point E gives a higher real wage compared to point P. The employment rate (capacity utilization rate) in E is also high compared to its counterpart because u in E is higher than in P. On the contrary, in the right figure, a higher real wage is given in point P compared to point E. However, the employment rate in P is low compared to that in E because u in P is smaller than u in E.

4.2 The case of the stagnationist regime

We consider the case where the steady state equilibrium exhibits the stagnationist regime.

First, we consider the case where the reserve army effect is small and consequently, the steady state equilibrium is stable. Figure 6 shows the phase diagram in the stagnationist regime case.

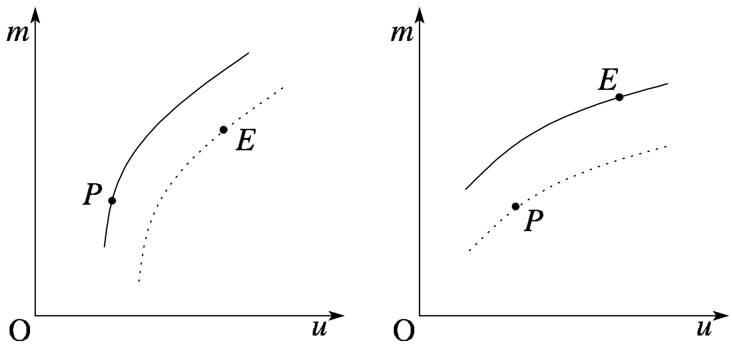

Figure 5: Iso-real wage curves and the two equilibria

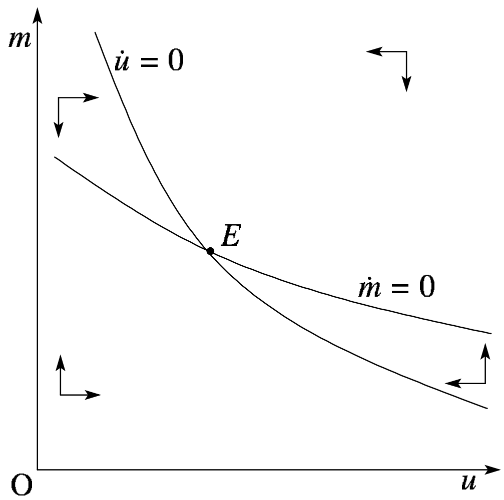

Figure 6: Phase diagram in the case where the stable steady state equilibrium exhibits the stagnationist regime

Next, we consider the case where the reserve army effect is large and consequently, the steady state equilibrium is unstable. Steady state E is the saddle point in Figure 7. Thus, if the economy starts from the initial value above the stable arm, for instance, point A_0, it converges to a point where the capacity utilization rate is zero and the profit share is unity. If the economy starts from the initial value below the stable arm, for instance, point B_0, it converges to a point where the capacity utilization rate is unity and the profit share is zero.

Here, we introduce $m_{\max}$ and $m_{\min}$. If the government does not actually know the steady state equilibrium, the minimum wage given by the government does not necessarily coincide with the steady state profit share. The interval $[m_{\min}, m_{\max}]$ represents the feasible combinations of the profit share determined by the government as the minimum wage policy. Then, as shown in Figure 7, starting from point A_0, the economy converges to point A, while starting from point B_0, it converges to point B. If we compare point A with point B, point B is superior to point A because the real wage associated with B is higher than that associated with A. Further, the employment rate in B is also high compared to that in A because u in B is higher than u in A.

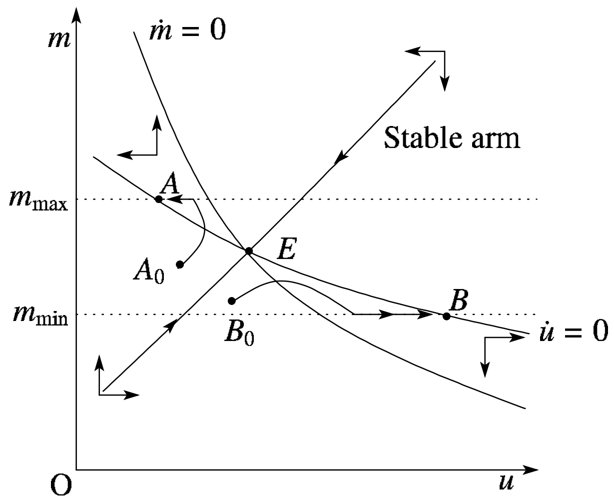

Figure 7: Phase diagram in the case where the unstable steady state equilibrium exhibits the stagnationist regime

5 Concluding remarks

In this paper, we have developed a Kaleckian model in which two types of labor (regular and non-regular employments) are incorporated. We have analyzed how the expansion of the wage gap between regular and non-regular employments affects the economy.

First, if the steady state equilibrium exhibits the stagnationist regime, an increase in the wage gap does not affect the stability of equilibrium. In this case, the size of the reserve army effect affects the stability of the equilibrium. If the reserve army effect is strong, the steady state equilibrium will be unstable. However, even if the steady state equilibrium is unstable, the introduction of the minimum and/or maximum wage prevents the capacity utilization and profit share from diverging. In this case, it is possible that depending on conditions, we obtain the real wage and the employment rate that are higher than the steady state equilibrium values.

Second, if the steady state equilibrium exhibits the exhilarationist regime, an increase in the wage gap destabilizes the equilibrium. It is possible that depending on conditions, an increase in the wage gap produces endogenous and perpetual business cycles. In addition, an increase in the non-regular employment relative to the regular employment stabilizes the steady state equilibrium. However, we must note that the government should not increase the non-regular employment literally. On the contrary, the government should adopt a policy mix that decreases both the non-regular employment and the wage gap. The introduction of the minimum wage is desirable in that it mitigates fluctuations of business cycles. However, the introduction of an inappropriate minimum wage policy that sets a minimum wage higher than the steady state equilibrium value consequently leads to the real wage and employment rate lower than the steady state values.

Finally, we must note that our model is a short-run one, and not a long-run one. As stated in the text, if we assume Okun's law, the capacity utilization rate has a one-to-one relationship with the employment rate. However, these two variables are strictly different. In this paper, for convenience, we identify the capacity utilization rate with the employment rate, but we cannot in a strict sense. To build a long-run model that distinguishes between the capacity utilization rate and the employment rate and investigate the employment in detail will be left for future research.

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