Kyoto University,
Graduate School of Economics
Research Project Center Discussion Paper Series

Trade Patterns and Non-Scale Growth between Two Countries

Hiroaki Sasaki

Discussion Paper No. E-12-006

Research Project Center
Graduate School of Economics
Kyoto University
Yoshida-Hommachi, Sakyo-ku
Kyoto City, 606-8501, Japan

August 2012

Trade Patterns and Non-Scale Growth between Two Countries

Hiroaki Sasaki*

Abstract

This paper builds a two-country, two-sector (manufacturing and agriculture), non-scale growth model and investigates the relationship between trade patterns and the growth rate of per capita real consumption. If the population growth rate of the home country is higher than that of the foreign country, the following results are obtained. (1) Under autarky, the growth rate of per capita real consumption is higher in the home country than in the foreign country. (2) Under free trade, if the home country completely specializes in manufacturing and the foreign country asymptotically completely specializes in agriculture, then the growth rate of the foreign country is higher than that of the home country, though this trade pattern is not sustainable in the long run. (3) Under free trade, if the home country produces both goods and the foreign country asymptotically completely specializes in agriculture, then the growth rates of the home country and the foreign country are equal, and this trade pattern is sustainable in the long run.

Keywords: non-scale growth model; trade patterns; population growth; per capita growth

JEL Classification: F10; F43; O11; O41

*Graduate School of Economics, Kyoto University. Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: sasaki@econ.kyoto-u.ac.jp. Phone: +81-(0)75-753-3446

1 Introduction

This paper builds a two-country, two-sector, non-scale growth model and investigates the relationship between trade patterns and economic growth. We investigate how the per capita growth rate of a country changes depending on the sector in which the country specializes.

Some other studies analyze the relationship between trade patterns and growth.[1] Kaneko (2000) builds a growth model with human capital accumulation and shows that the relationship between the terms of trade and growth depends on whether the country specializes in the consumption goods sector or the investment goods sector. If the home country specializes in the investment goods sector, its growth rate does not depend on the terms of trade. On the other hand, if the home country specializes in the consumption goods sector, its growth rate does depend on the terms of trade and increases as the terms of trade improve. However, Kaneko's (2000) model is a small-open-economy model, and hence, the terms of trade are given exogenously.

Kaneko (2003) builds a two-country, two-sector, AK growth model and endogenizes the terms of trade. He finds that if a country with a lower growth rate than its trade partner under autarky has a comparative advantage in the consumption goods sector, then the country can narrow or even reverse the growth gap by opening trade.

Felbermayr (2007) describes a situation where a capital-abundant North and a capital-scarce South trade with each other. In his model, the trade pattern is endogenously determined, and he analyzes the situation where the North produces investment goods and the South produces consumption goods. The production technology of investment goods is AK and that of consumption goods is decreasing returns to scale. Along the balanced growth path, the Southern terms of trade are continuously improving such that even the decreasing-returns-to-scale South can grow at the same rate as the North. Therefore, the South can eliminate the growth gap by opening trade.

The above studies use scale-growth models. That is, population size positively affects per capita growth. This assumption, however, seems counterfactual. Jones (1995) attempts to remove the scale effects and presents a non-scale growth model in which the growth rate of output per capita depends positively on the population growth rate and not on the size of the population. That is, the higher the population growth rate, the faster the country grows.[2]

[1] Wong and Yip (1999) present a small-open-economy, two-sector model of endogenous growth with capital accumulation and learning by doing and analyze the relationship between economic growth, industrialization, and international trade.

[2] For a systematic exposition of scale effects and non-scale growth, see Jones (1999). For more sophisticated non-scale growth models, see also Kortum (1997), Dinopoulos and Thompson (1998), Peretto (1998), Segerstrom (1998), Young (1998), and Howitt (1999).

In this paper, we build a two-country, two-sector, non-scale growth model in which manufacturing has increasing returns to scale and agriculture has constant returns to scale. We investigate the relationship between trade patterns and the growth and income gaps between the two countries under free trade in the long run.[3]

We use the non-scale growth model for two reasons. First, we can obtain sustainable income per capita growth even though population growth is strictly positive, and second, we do not need to impose knife-edge conditions on the parameters of the model. To our knowledge, this model differs from almost all other models in that we explicitly consider population growth. In addition, existing models belong to the AK class of models, and as such, impose knife-edge conditions on the production functions.

According to the analysis, we find that the difference between the population growth rates of the two countries affects the trade patterns and the relationship between the per capita growth of the home country (Home, hereafter) and that of the foreign country (Foreign, hereafter). In this paper, without loss of generality, we assume that the population growth rate of Home is greater than or equal to that of Foreign.

We first discuss the case where Home and Foreign have equal population growth rates. Under autarky, the per capita consumption growth rates of the two countries are equal. Under free trade, two trade patterns may be realized in the long run. First, Home diversifies while Foreign asymptotically completely specializes in agriculture along the balanced growth path (BGP, hereafter).[4] This trade pattern is sustainable, which means that if an economy is located on the BGP, then it continues on the BGP over time. Second, Home completely specializes in manufacturing while Foreign asymptotically completely specializes in agriculture along the BGP. This trade pattern is also sustainable. If population growth is equal in the two countries, then under both autarky and free trade, per capita consumption growth is equal.

Next, we discuss the case where the population growth rate is larger in Home than in Foreign. Under autarky, per capita consumption in Home is larger than in Foreign. Under free trade, two trade patterns may be realized. First, Home diversifies while Foreign asymptotically completely specializes in agriculture along the BGP. This trade pattern is sustainable. In this case, per capita consumption growth is equal in the two countries and is the same as per capita consumption growth in Home under autarky. Therefore, Foreign's growth rate under free trade is higher than under autarky, and the growth gap under autarky vanishes.

[3]Our model is based on the work of Christiaans (2008). He extends Wong and Yip's (1999) model and develops a small-open-economy, non-scale growth model in which agriculture has constant returns to scale and manufacturing has increasing returns to scale and examines the dynamics as the economy moves toward the long-run equilibrium.

[4]The word "asymptotically" means that the agricultural output converges to zero, but it never vanishes because we assume that Foreign's capital stock is strictly positive. See also Christiaans (2008).

Second, Home completely specializes in manufacturing while Foreign asymptotically completely specializes in agriculture along the BGP. However, this trade pattern is not sustainable. While Foreign is on the BGP, its per capita consumption growth exceeds that of Home. Therefore, the growth gap under autarky is reversed. However, as stated above, this trade pattern eventually becomes unsustainable and shifts to the first trade pattern. At this point, the growth rates of per capita consumption in both countries are equal.

The rest of the paper is organized as follows. Section 2 presents the framework of the model and analyzes the equilibrium under autarky. Section 3 presents a free trade equilibrium corresponding to each trade pattern and investigates whether each trade pattern is sustainable. Section 4 compares the growth rates of per capita real consumption under autarky and free trade in both countries. Section 5 concludes the paper.

2 The model

Consider a world that consists of two countries: Home and Foreign. Both countries produce homogeneous manufactured and agricultural goods. The manufactured good is used for both consumption and investment whereas the agricultural good is used only for consumption.

2.1 Production

Firms produce manufactured goods X_i^M with labor input L_i^M and capital stock K_i and produce agricultural goods X_i^A with only labor input L_i^A. Here, $i = 1$ and $i = 2$ denote Home and Foreign, respectively. Both countries have the same production functions, which are specified as follows:

$$X_i^M = A_i K_i^{\alpha} (L_i^M)^{1-\alpha}, \quad \text{where } A_i = K_i^{\beta} \tag{1}$$

$$= K_i^{\alpha+\beta} (L_i^M)^{1-\alpha}, \quad 0 < \alpha < 1,\ 0 < \beta < 1,\ \alpha + \beta < 1, \tag{2}$$

$$X_i^A = L_i^A. \tag{3}$$

Here, A_i in equation (1) represents an externality associated with capital accumulation, which captures the learning-by-doing effect à la Arrow (1962). Substituting A_i into equation (1), we obtain equation (2), which shows that manufactured goods production has increasing returns to scale with β corresponding to the extent of the increasing returns. Equation (3) shows that agricultural goods production has constant returns to scale.

Suppose that labor supply is equal to the population and that the population is fully employed. Moreover, suppose that the population grows at a constant rate n_i and the initial population is unity in each country: $L_i(t) = L_i^M(t) + L_i^A(t) = e^{n_i t}$, $n_i > 0$.

Let p_i denote the price of manufactured goods relative to agricultural goods. Then, the profits of manufacturing and agricultural firms are given by $\pi_i^M = p_i X_i^M - w_i L_i^M - p_i r_i K_i$ and $\pi_i^A = X_i^A - w_i L_i^A$, respectively, where w_i denotes the wage in terms of agricultural goods and r_i denotes the rental rate of capital.

From the profit-maximizing conditions, we obtain the following relations:

$$p_i \frac{\partial X_i^M}{\partial L_i^M} = w_i = 1, \tag{4}$$

$$\frac{\partial X_i^M}{\partial K_i} = r_i \text{ with } A_i \text{ given.} \tag{5}$$

From equation (4), we find that the wage is unity as long as agricultural production is positive. We assume a Marshallian externality in deriving equation (5); profit-maximizing firms regard A_i as exogenously given. Accordingly, firms do not internalize the effect of A_i.

2.2 Consumption

For simplification, we make the classical assumption that wage income and capital income are entirely devoted to consumption and saving, respectively.[5] In the canonical one-sector Solow model, under the golden rule steady state where per capita consumption is maximized, consumption is equal to the real wage and all profits are saved and invested. Hence, our assumption has some rationality and can be interpreted as a simple rule of thumb for consumers with dynamic optimization (Christiaans, 2008). We define real consumption per capita c_i as $c_i = C_i/L_i = (C_i^M)^\gamma (C_i^A)^{1-\gamma}/L_i$, where C_i denotes economy-wide real consumption. In this case, a fraction γ of wage income is spent on C_i^M and the rest $1-\gamma$ is spent on C_i^A.

$$p_i C_i^M = \gamma w_i L_i, \tag{6}$$

$$C_i^A = (1-\gamma) w_i L_i. \tag{7}$$

Moreover, the following relationship between real investment I_i and saving holds: $p_i I_i = p_i r_i K_i$. From this equation, we obtain the rate of capital accumulation:

$$\frac{\dot{K}_i}{K_i} = r_i. \tag{8}$$

That is, the rate of capital accumulation is equal to the rental rate of capital. A dot over a variable denotes the time derivative of the variable (e.g., $\dot{K}_i \equiv dK_i/dt$).

[5] The same assumption is also used in Krugman (1981), which considers a two-country, two-sector, North-South trade and development model.

2.3 Equilibrium under autarky

Under autarky, both goods have to be produced. The market-clearing conditions are as follows: $X_i^M = C_i^M + I_i$ and $X_i^A = C_i^A$. Note that $w_i = 1$ under autarky. From the market-clearing condition for manufactured goods, we obtain p_i, which is used to derive each sector's employment share: $L_i^M/L_i = \gamma$ and $L_i^A/L_i = 1-\gamma$. Therefore, under autarky, each sector's employment share is constant.

Under autarky, the relative price of manufactured goods is given by

$$p_i = \frac{(\gamma L_i)^\alpha}{(1-\alpha)K_i^{\alpha+\beta}}. \tag{9}$$

We now derive the BGP under autarky. Along the BGP, the rate of capital accumulation is constant and equal to the rental rate of capital, which is given from equation (5) by $r_i = \alpha K_i^{\alpha+\beta-1}(\gamma L_i)^{1-\alpha}$. From this, the BGP growth rates of K_i and p_i are, respectively, given by

$$g_{K_i}^* = \frac{1-\alpha}{1-\alpha-\beta} n_i > 0, \tag{10}$$

$$g_{p_i}^* = -\frac{\beta}{1-\alpha-\beta} n_i < 0, \tag{11}$$

where $g_x \equiv \dot{x}/x$ denotes the growth rate of a variable x and an asterisk "$*$" denotes a BGP value. The rate of capital accumulation is positive and proportionate to population growth, and the relative price of manufactured goods is decreasing at a constant rate.[6]

3 Equilibrium under free trade

Suppose that Home and Foreign engage in free trade at time zero. If $K_1(0) > K_2(0)$, then from equation (9), $p_1(0) < p_2(0)$ because $L_1(0) = L_2(0) = 1$. Thus, if $K_1(0) > K_2(0)$, Home has a comparative advantage in manufactured goods and Foreign has a comparative advantage in agricultural goods. In the following analysis, we assume that $K_1(0) > K_2(0)$.

[6] Considering the BGP growth rate of capital stock, we introduce a new variable, scale-adjusted capital stock: $k_i \equiv K_i/L_i^\phi$. The dynamics of the scale-adjusted capital stock are given by

$$\dot{k}_i = \alpha\gamma^{1-\alpha}k_i^{\alpha+\beta} - \phi n_i k_i, \quad \text{where } \phi \equiv \frac{1-\alpha}{1-\alpha-\beta}.$$

In the steady state, $\dot{k}_i = 0$, from which we obtain

$$k_i^* = \{[\alpha\gamma^{1-\alpha}/(\phi n_i)]\}^{\frac{1}{1-\alpha-\beta}}.$$

The steady state is stable because $d\dot{k}_i/k_i|_{k_i=k_i^*} = -k_i^*[(1-\alpha-\beta)\alpha\gamma^{1-\alpha}(k_i^*)^{\alpha+\beta-2} + \phi n_i] < 0$.

It is sufficient for our purpose to consider the following four trade patterns from the viewpoint of Home:[7]

Case 1 : Both countries produce both goods, that is, both countries diversify.
Case 2 : Home diversifies and Foreign completely specializes in agriculture.
Case 3 : Home completely specializes in manufacturing and Foreign completely specializes in agriculture.
Case 4 : Home completely specializes in manufacturing and Foreign diversifies.

3.1 Equilibrium when both countries diversify—Case 1

The market-clearing conditions for both goods are given by

$$X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2, \tag{12}$$
$$X_1^A + X_2^A = C_1^A + C_2^A. \tag{13}$$

From these, we obtain

$$p^{\frac{1}{\alpha}} = \frac{\gamma(L_1 + L_2)}{(1-\alpha)^{\frac{1}{\alpha}}\left(K_1^{\frac{\alpha+\beta}{\alpha}} + K_2^{\frac{\alpha+\beta}{\alpha}}\right)}. \tag{14}$$

Each country's employment share of the manufacturing sector θ_i^M is given by

$$\theta_1^M \equiv \frac{L_1^M}{L_1} = \frac{\gamma\left(1 + \frac{L_2}{L_1}\right)}{1 + \left(\frac{K_2}{K_1}\right)^{\frac{\alpha+\beta}{\alpha}}}, \tag{15}$$
$$\theta_2^M \equiv \frac{L_2^M}{L_2} = \frac{\gamma\left(1 + \frac{L_1}{L_2}\right)}{1 + \left(\frac{K_1}{K_2}\right)^{\frac{\alpha+\beta}{\alpha}}}. \tag{16}$$

The rates of capital accumulation in both countries are given by

$$\frac{\dot{K}_1}{K_1} = \alpha K_1^{\alpha+\beta-1}(\theta_1^M L_1)^{1-\alpha}, \tag{17}$$
$$\frac{\dot{K}_2}{K_2} = \alpha K_2^{\alpha+\beta-1}(\theta_2^M L_2)^{1-\alpha}. \tag{18}$$

[7] In Sasaki (2011a), a detailed analysis with the use of phase diagrams is conducted under the assumption that $n_1 = n_2 = n$.

First, if $n_1 = n_2$, so that $L_1 = L_2$,[8] then, after enough time has passed, we obtain

$$\lim_{t\to+\infty} \theta_1^M = 2\gamma, \tag{19}$$

$$\lim_{t\to+\infty} \theta_2^M = 0, \tag{20}$$

where $\gamma < 1/2$. The manufacturing employment share in Foreign goes to zero, and Foreign asymptotically completely specializes in agriculture. Hence, Case 1 is not sustainable when $n_1 = n_2$.

Next, if $n_1 > n_2$, after enough time has passed, we obtain

$$\lim_{t\to+\infty} \theta_1^M = \gamma, \tag{21}$$

$$\lim_{t\to+\infty} \theta_2^M = 0. \tag{22}$$

In this case too, the manufacturing employment share in Foreign goes to zero, and Foreign asymptotically completely specializes in agriculture. Hence, Case 1 also is unsustainable when $n_1 > n_2$.

Figures 1 to 6 show a numerical example in which $\alpha = 0.3, \beta = 0.2, \gamma = 0.6, K_1(0) = 1.2, K_2(0) = 1, n_1 = 0.02$, and $n_2 = 0.01$.

[Figures 1 to 6 around here]

The above analysis shows that Case 1 is unsustainable irrespective of the size of population growth.

3.2 Equilibrium when Home diversifies and Foreign specializes in agriculture—Case 2

The market-clearing conditions for both goods are given by

$$X_1^M = C_1^M + C_2^M + I_1, \tag{23}$$

$$X_1^A + X_2^A = C_1^A + C_2^A. \tag{24}$$

Hence, we obtain

$$p^{\frac{1}{\alpha}} = \frac{\gamma(L_1 + L_2)}{(1-\alpha)^{\frac{1}{\alpha}} K_1^{\frac{\alpha+\beta}{\alpha}}}. \tag{25}$$

[8]In addition, if $K_1(0) = K_2(0)$, the manufacturing employment share in each country is given by $\theta_i^M = \gamma$, which is constant. Case 1 is only sustainable in this case. However, the relative prices in both countries under autarky are equal, and therefore trade does not occur.

The manufacturing employment share in Home is given by

$$\theta_1^M = \gamma\left(1 + \frac{L_2}{L_1}\right). \tag{26}$$

If $n_1 = n_2 = n$, the manufacturing employment share in Home becomes

$$\theta_1^M = 2\gamma. \tag{27}$$

In this case, we need $\gamma < 1/2$ for Case 2 to hold.

On the other hand, if $n_1 > n_2$, we obtain

$$\lim_{t \to +\infty} \theta_1^M = \gamma. \tag{28}$$

The manufacturing employment share of Home converges to γ.

The growth rate of capital stock is given by

$$\frac{\dot{K}_1}{K_1} = \alpha\gamma^{1-\alpha}(L_1 + L_2)^{1-\alpha} K_1^{\alpha+\beta-1}. \tag{29}$$

The rate of change of capital stock growth rate is given by

$$\frac{\dot{g}_{K_1}}{g_{K_1}} = \begin{cases} (1-\alpha)n + (\alpha+\beta-1)g_{K_1} & \text{if } n_1 = n_2 = n, \\ (1-\alpha)\left(\frac{L_1}{L_1+L_2} n_1 + \frac{L_2}{L_1+L_2} n_2\right) + (\alpha+\beta-1)g_{K_1} & \text{if } n_1 > n_2. \end{cases} \tag{30}$$

When $n_1 > n_2$ we obtain $L_1/(L_1 + L_2) \to 1$ and $L_2/(L_1 + L_2) \to 0$ after enough time has passed, and hence,

$$\frac{\dot{g}_{K_1}}{g_{K_1}} = (1-\alpha)n_1 + (\alpha+\beta-1)g_{K_1}. \tag{31}$$

From this, the BGP growth rate of capital stock and the terms of trade become

$$g_{K_1}^* = \frac{1-\alpha}{1-\alpha-\beta} n_1 > 0, \tag{32}$$

$$g_p^* = -\frac{\beta}{1-\alpha-\beta} n_1 < 0. \tag{33}$$

Along the BGP, the terms of trade continue to decrease at a constant rate.

We now examine the conditions under which Case 2 holds in detail. Following Wong and Yip (1999), we investigate whether or not the trade pattern is sustainable by comparing the size of the terms of trade and that of the marginal rate of transformation (MRT) of the production possibilities frontier (PPF) at the corner point where a country completely specializes in manufacturing.

First, the size of the MRT of PPF in Home is given by

$$-\frac{dX_1^A}{dX_1^M} = \frac{[K_1^{\alpha+\beta}(\theta_1^M L_1)^{1-\alpha}]^{\frac{\alpha}{1-\alpha}}}{(1-\alpha)K_1^{\frac{\alpha+\beta}{1-\alpha}}}. \tag{34}$$

Second, substituting $\theta_1^M = 1$ in equation (34), the size of the MRT at the point where Home completely specializes in manufacturing is given by

$$\bar{\chi}_1 = \frac{L_1^\alpha}{(1-\alpha)K_1^{\alpha+\beta}}. \tag{35}$$

[Figure 7 around here]

For Case 2 to be sustainable, we need $p < \bar{\chi}_1$.[9] Rearranging this condition, we obtain

$$L_2 < \frac{1-\gamma}{\gamma} L_1. \tag{36}$$

If $n_1 = n_2$, that is, $L_1 = L_2$, this condition simplifies to $\gamma < 1/2$. Therefore, if the expenditure share of manufactured goods is less than 1/2, Case 2 is sustainable.

If $n_1 > n_2$, then $L_1 > L_2$. If $\gamma < 1/2$, we obtain $(1-\gamma)/\gamma > 1$, and hence, condition (36) holds. If $\gamma > 1/2$, we obtain $(1-\gamma)/\gamma < 1$. In this case, if L_1 is sufficiently large, condition (36) holds. In fact, with $n_1 > n_2$, L_1 becomes larger than L_2 after some time has passed. Therefore, if $n_1 > n_2$, Case 2 is sustainable regardless of the size of γ.

Note that in this case, we obtain $c_1 = c_2$ because $w_1 = w_2 = 1$ and both countries face the same relative price p.

3.3 Equilibrium when Home specializes in manufacturing and Foreign specializes in agriculture—Case 3

The market-clearing conditions for both goods are given by

$$X_1^M = C_1^M + C_2^M + I_1, \tag{37}$$

$$X_2^A = C_1^A + C_2^A. \tag{38}$$

With $L_1^M = L_1$, we obtain

$$p = \frac{\gamma L_2}{(1-\alpha)(1-\gamma)K_1^{\alpha+\beta}L_1^{1-\alpha}}. \tag{39}$$

[9] Appendix A explains why Foreign cannot completely specialize in manufacturing.

The growth rate of capital stock is given by

$$\frac{\dot{K}_1}{K_1} = \alpha K_1^{\alpha+\beta-1} L_1^{1-\alpha}. \tag{40}$$

The BGP growth rate of Home's capital stock becomes

$$g^*_{K_1} = \frac{1-\alpha}{1-\alpha-\beta} n_1 > 0. \tag{41}$$

In addition, from equation (39), the growth rate of the terms of trade is given by

$$g^*_p = \begin{cases} -\dfrac{\beta}{1-\alpha-\beta} n < 0 & \text{if } n_1 = n_2 = n, \\ n_2 - \dfrac{1-\alpha}{1-\alpha-\beta} n_1 = -\dfrac{(1-\alpha)(n_1-n_2)+\beta n_2}{1-\alpha-\beta} < 0 & \text{if } n_1 > n_2. \end{cases} \tag{42}$$

Hence, along the BGP, the relative price of manufactured goods continues to decrease at a constant rate.

Case 3 is sustainable if $\bar{\chi}_1 < p$, which can be rewritten as

$$L_1 < \frac{\gamma}{1-\gamma} L_2. \tag{43}$$

First, if $n_1 = n_2$, that is, $L_1 = L_2$, this condition simplifies to $\gamma > 1/2$. Therefore, if the population growth rates in both countries are equal, Case 3 is sustainable as long as the expenditure share of manufactured goods is more than 1/2.

Second, we consider the case where $n_1 > n_2$, that is, $L_1 > L_2$. If $\gamma < 1/2$, we obtain $0 < \gamma/(1-\gamma) < 1$, and hence, condition (43) is not satisfied. If $\gamma > 1/2$, we obtain $\gamma/(1-\gamma) > 1$, and hence, condition (43) can be satisfied even though $L_1 > L_2$. However, as the time passes, L_1 becomes sufficiently larger than L_2 and condition (43) will be violated. Thus, if the population growth rate is larger in Home than in Foreign, Case 3 is sustainable as long as the expenditure share for manufactured goods is greater than 1/2, but it will be unsustainable in the long run. Once Case 3 becomes unsustainable, it will switch to Case 2.[10]

Note that in this case, we obtain $c_1 > c_2$ because $w_1 = [\gamma/(1-\gamma)] \cdot (L_2/L_2) > 1 > w_2 = 1$ and both countries face the same relative price p.

3.4 Equilibrium when Home specializes in manufacturing and Foreign diversifies—Case 4

The market-clearing conditions for both goods are given by

$$X_1^M + X_2^M = C_1^M + C_2^M + I_1 + I_2, \tag{44}$$

$$X_2^A = C_1^A + C_2^A. \tag{45}$$

[10] Appendix B explains why Case 3 switches to Case 2.

From these equations, we find that the terms of trade satisfy the following equation:

$$(1-\alpha)^{\frac{1}{\alpha}} p^{\frac{1}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}} = \gamma L_2 - (1-\alpha)(1-\gamma) p K_1^{\alpha+\beta} L_1^{1-\alpha}. \tag{46}$$

From this equation, p is implicitly and uniquely determined, and hence, p is a function of K_1, K_2, L_1, and L_2: $p = p(K_1, K_2, L_1, L_2)$.[11]

The growth rate of the capital stock in each country is given by

$$\frac{\dot{K}_1}{K_1} = \alpha K_1^{\alpha+\beta-1} L_1^{1-\alpha}, \tag{47}$$

$$\frac{\dot{K}_2}{K_2} = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} p^{\frac{1-\alpha}{\alpha}} K_2^{\frac{\beta}{\alpha}}, \tag{48}$$

where p is endogenously determined by equation (46).

The employment share of manufacturing in Foreign is given by

$$\theta_2^M = \frac{(1-\alpha)^{\frac{1}{\alpha}} p^{\frac{1}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}}}{L_2}. \tag{49}$$

In this case, analytical solutions are hard to obtain, so we conduct numerical simulations. Using equation (46), we obtain the time derivative of p as follows:

$$\dot{p} = \frac{\gamma \dot{L}_2 - (1-\alpha)^2(1-\gamma) p K_1^{\alpha+\beta} L_1^{-\alpha} \dot{L}_1 - (1-\alpha)(1-\gamma)(\alpha+\beta) p K_1^{\alpha+\beta-1} L_1^{1-\alpha} \dot{K}_1 - \frac{\alpha+\beta}{\alpha}(1-\alpha)^{\frac{1}{\alpha}} K_2^{\frac{\beta}{\alpha}} \dot{K}_2}{\frac{1}{\alpha}(1-\alpha)^{\frac{1}{\alpha}} p^{\frac{1-\alpha}{\alpha}} K_2^{\frac{\alpha+\beta}{\alpha}} + (1-\alpha)(1-\gamma) K_1^{\alpha+\beta} L_1^{1-\alpha}}. \tag{50}$$

Substituting equations (47) and (48) into equation (50), we obtain the differential equation of p.

With initial conditions $K_1(0)$, $K_2(0)$, $L_1(0)$, $L_2(0)$, and the parameters, we can obtain the initial value of the terms of trade, $p(0)$, using equation (46). Using this initial value $p(0)$ and equation (50), we obtain the time path of $p(t)$.

From the numerical simulation, we find that regardless of whether $n_1 > n_2$ or $n_1 = n_2$, the manufacturing employment share in Foreign tends to zero in finite time, that is, $\theta_2^M \to 0$ (see Figures 7 to 10). Therefore, Case 4 is unsustainable in the long run.

[Figures 7 to 10 around here]

If p is extremely large when switching from autarky to free trade, Home completely specializes in manufacturing while Foreign diversifies. However, since p continues to decrease over time, Foreign asymptotically completely specializes in agriculture (Case 3).

[11] The left-hand side of equation (46) is an increasing function of p, whereas the right-hand side of this equation is a decreasing function of p. Plotting both functions, we find that the intersection of the functions is unique and gives an instantaneous equilibrium value of p.

4 Per capita consumption growth rates

Consumption is defined as wages only, and hence, the growth rate of per capita real consumption is equal to the growth rate of the real wage.

$$g_{c_i} = g_{w_i} - \gamma g_{p_i}. \tag{51}$$

Here, the real wage is deflated by the consumer price index p_i^{γ}.[12]

To obtain g_{c_i}, we must determine the growth rate of the nominal wage and that of the terms of trade. Note that as long as agricultural goods are produced, the nominal wage is equal to unity, that is, $w_i = 1$, which means that $g_{w_i} = 0$ as long as $X_i^A > 0$. However, when a country completely specializes in manufacturing under free trade, we obtain $w_i = [\gamma/(1-\gamma)] \cdot (L_2/L_1) > 1$, which means that $g_{w_i} = n_2 - n_1 < 0$.

4.1 Autarky

Under autarky, the growth rate of per capita consumption is given by

$$g_{c_i}^{AT} = \frac{\gamma\beta}{1-\alpha-\beta} n_i > 0, \tag{52}$$

where AT denotes "autarky." From this, if $n_1 = n_2$, we obtain $g_{c_1}^{AT} = g_{c_2}^{AT}$, and if $n_1 > n_2$, we obtain $g_{c_1}^{AT} > g_{c_2}^{AT}$.

4.2 Case 2

When Home diversifies and Foreign asymptotically completely specializes in agriculture, the growth rates of per capita consumption are equal and are given by

$$g_{c_1}^{FT_2} = g_{c_2}^{FT_2} = \begin{cases} \dfrac{\gamma\beta}{1-\alpha-\beta} n > 0 & \text{if } n_1 = n_2 = n, \\ \dfrac{\gamma\beta}{1-\alpha-\beta} n_1 > 0 & \text{if } n_1 > n_2, \end{cases} \tag{53}$$

where FT_2 denotes "free trade" in Case 2.

4.3 Case 3

When both countries completely specialize, if $n_1 = n_2 = n$, both countries' growth rates are given by

$$g_{c_1}^{FT_3} = g_{c_2}^{FT_3} = \frac{\gamma\beta}{1-\alpha-\beta} n > 0. \tag{54}$$

[12]Strictly speaking, the consumer price index is given by $\gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} p_i^{\gamma}$. However, we use p_i^{γ} because the constant terms have no effect on the results.

On the other hand, if $n_1 > n_2$, both countries' growth rate are given by

$$g_{c_1}^{FT_3} = \frac{\beta - (1-\gamma)(1-\alpha)}{1-\alpha-\beta} n_1 + (1-\gamma) n_2, \tag{55}$$

$$g_{c_2}^{FT_3} = \frac{\gamma(1-\alpha)}{1-\alpha-\beta} n_1 - \gamma n_2 > 0. \tag{56}$$

$g_{c_2}^{FT_3} > 0$ necessarily holds because the condition under which $g_{c_2}^{FT3} > 0$ becomes

$$n_2 < \frac{1-\alpha}{1-\alpha-\beta} n_1, \tag{57}$$

which necessarily holds because $n_1 > n_2$.

The coefficient on n_1 in $g_{c_1}^{FT_3}$ can be positive or negative. Indeed, under the restriction of $\alpha + \beta < 1$, the term $\beta - (1-\gamma)(1-\alpha)$ can be positive or negative. Therefore, an increase in the population growth in Home can either increase or decrease the growth rate of per capita consumption in Home.

We consider the conditions under which $g_{c_1}^{FT_3} > 0$ holds. If the coefficient on n_1 is positive, we always have $g_{c_1}^{FT_3} > 0$. Even if the coefficient on n_1 is negative, there exists a combination of n_1 and n_2 that satisfies $n_1 > n_2$ and $g_{c_1^{FT_3}} > 0$:

$$n_2 > \frac{(1-\gamma)(1-\alpha) - \beta}{(1-\gamma)(1-\alpha-\beta)} n_1. \tag{58}$$

These results are similar to those obtained by Sasaki (2011b). He builds a non-scale growth, North-South economic development model and shows that along the BGP, both countries grow at the same rate but per capita incomes grow at different rates because of the differences in population growth. In Sasaki (2011b), the growth rate of per capita consumption in the North may be increasing or decreasing in Northern population growth but is increasing in Southern population growth, and the growth rate of per capita consumption in the South is decreasing in Southern population growth but is increasing in Northern population growth. We obtain similar results with the North corresponding to Home and the South corresponding to Foreign.

However, this paper does differ from Sasaki (2011b) in two ways. First, in Sasaki (2011b), the production pattern is fixed and given exogenously, and population growth in the North is lower than that in the South, which corresponds to $n_1 < n_2$ in this paper. The low-population-growth North produces only manufactured goods, whereas the high-population-growth South produces only agricultural goods. However, in our model, the trade patterns are endogenously determined, so that the country with high population growth specializes in manufacturing and the country with low population growth specializes in agriculture in the long run.

Moreover, in Sasaki (2011b), if $n_1 < n_2$, we obtain $g_{c_1} > g_{c_2}$. That is, the growth rate of per capita consumption in the low-population-growth manufacturing country is higher than that of the high-population-growth agricultural country. In this paper, by contrast, if $n_1 > n_2$, we obtain $g_{c_1}^{FT_3} < g_{c_2}^{FT_3}$. That is, the per capita growth rate of the high-population-growth manufacturing country is lower than that of the low-population-growth agricultural country.

4.4 Comparison between autarky and free trade

In this subsection, we assume $n_1 > n_2$.

Under autarky, we have $g_{c_1}^{AT} - g_{c_2}^{AT} = n_1 - n_2 > 0$, and hence, there exists a growth gap.

Proposition 1. *Suppose that population growth in Home is higher than in Foreign. Then, under autarky, the BGP growth rate of per capita consumption in Home is higher than in Foreign.*

When Home diversifies and Foreign asymptotically completely specializes in agriculture (i.e., Case 2), we have $g_{c_1}^{FT_2} - g_{c_2}^{FT_2} = 0$, and hence, the growth gap disappears.

Proposition 2. *Suppose that when switching from autarky to free trade, both countries diversify. Then, in the long run, Home diversifies and Foreign asymptotically completely specializes in agriculture. This situation is sustainable. Moreover, the growth rates and levels of per capita consumption in both countries are equal.*

When Home completely specializes in manufacturing and Foreign asymptotically completely specializes in agriculture (i.e., Case 3), we have $g_{c_1}^{FT_3} - g_{c_2}^{FT_3} = -(n_1 - n_2) < 0$, and hence, there exists a growth gap.

Proposition 3. *Suppose that when switching from autarky to free trade, Home completely specializes in manufacturing and Foreign asymptotically completely specializes in agriculture. This situation is sustainable for a while. Moreover, the growth rate of per capita consumption in Foreign is higher than that in Home, but the level of per capita consumption in Foreign is lower than that in Home.*

We now compare the growth rates under autarky with those under free trade. When Home completely specializes in manufacturing and Foreign asymptotically completely specializes in agriculture, we obtain the following relations:

$$g_{c_1}^{FT_3} - g_{c_1}^{AT} = -(1-\gamma)(n_1 - n_2) < 0, \tag{59}$$

$$g_{c_2}^{FT_3} - g_{c_2}^{AT} = \frac{\gamma(1-\alpha)}{1-\alpha-\beta}(n_1 - n_2) > 0. \tag{60}$$

From these, we have $g_{c_1}^{FT_3} < g_{c_1}^{AT}$ and $g_{c_2}^{FT_3} > g_{c_2}^{AT}$. Next, we obtain

$$g_{c_1}^{FT_3} - g_{c_2}^{AT} = \frac{\beta - (1-\gamma)(1-\alpha)}{1-\alpha-\beta}(n_1 - n_2), \tag{61}$$

$$g_{c_2}^{FT_3} - g_{c_1}^{AT} = \gamma(n_1 - n_2) > 0. \tag{62}$$

Accordingly, we have $g_{c_2}^{FT_3} > g_{c_1}^{AT}$. Moreover, if $\beta - (1-\gamma)(1-\alpha) > 0$, we have $g_{c_1}^{FT_3} > g_{c_2}^{AT}$, and if $\beta - (1-\gamma)(1-\alpha) < 0$, we have $g_{c_1}^{FT_3} < g_{c_2}^{AT}$.

From the above analysis, we obtain the following three sets of inequalities:

$$\text{Case 2}: g_{c_2}^{AT} < g_{c_1}^{AT} = g_{c_1}^{FT_2} = g_{c_2}^{FT_2}, \tag{63}$$

$$\text{Case 3a}: g_{c_2}^{AT} < g_{c_1}^{FT_{3a}} < g_{c_1}^{AT} < g_{c_2}^{FT_{3a}}, \quad \text{where } \beta - (1-\gamma)(1-\alpha) > 0, \tag{64}$$

$$\text{Case 3b}: g_{c_1}^{FT_{3b}} < g_{c_2}^{AT} < g_{c_1}^{AT} < g_{c_2}^{FT_{3b}}, \quad \text{where } \beta - (1-\gamma)(1-\alpha) < 0. \tag{65}$$

Therefore, we obtain the following two propositions:

Proposition 4. *Suppose that under free trade, in the long run, Home diversifies and Foreign asymptotically completely specializes in agriculture. The growth rate of per capita consumption in Home under free trade is equal to that under autarky, whereas the growth rate of per capita consumption in Foreign under free trade is higher than that under autarky.*

Proposition 5. *Suppose that under free trade, in the long run, Home completely specializes in manufacturing and Foreign asymptotically completely specializes in agriculture. The growth rate of per capita consumption in Home under free trade is lower than that under autarky, whereas the growth rate of per capita consumption in Foreign under free trade is higher than that under autarky. In addition, the growth rate of Home under free trade may be higher or lower than that of Foreign under autarky.*

In Case 2, agricultural production is positive and hence, $w_i = 1$ in both countries. In addition, the terms of trade are common to both countries. Therefore, the growth rate of per capita consumption is equal in both countries, that is, $g_{c_1}^{FT_2} = g_{c_2}^{FT_2}$.

In Case 3, the growth rate of the nominal wage is different in each country. In Home, $g_{w_1}^{FT_3} = -(n_1 - n_2) < 0$, whereas in Foreign, $g_{w_2}^{FT_3} = 0$. Therefore, the growth rate of per capita consumption in Home is less than that in Foreign, that is, $g_{c_1}^{FT_3} < g_{c_2}^{FT_3}$.

5 Conclusions

This paper has built a two-country, two-sector, non-scale growth model and has investigated the relationship between trade patterns and per capita growth.

If population growth is greater in Home than in Foreign, Home becomes a manufacturing country and Foreign becomes an agricultural country when switching from autarky to free trade. However, based only on this specialization pattern, we cannot say that Foreign becomes worse off because the per capita growth rate under free trade exceeds that under autarky.

In the case where Home completely specializes in manufacturing and Foreign asymptotically completely specializes in agriculture, the per capita growth rate of Foreign exceeds that of Home, although this trade pattern is unsustainable in the long run.

Therefore, in both cases, Foreign can be better off under free trade than under autarky.

Acknowledgments

I am grateful to Grants-in-Aid for Scientific Research of Japan Society for the Promotion of Science (KAKENHI 23730234) for financial support. The usual disclaimer applies.

Appendix

A Why Foreign's complete specialization in manufacturing is unsustainable

The slope of the PPF at the corner point where Foreign completely specializes in manufacturing is given by

$$\bar{\chi}_2 = \frac{L_2^{\alpha}}{(1-\alpha)K_2^{\alpha+\beta}}. \tag{A-1}$$

When Foreign specializes in agriculture, the growth rate of $\bar{\chi}_2$ is given by

$$g_{\bar{\chi}_2} = \alpha n_2 > 0. \tag{A-2}$$

Hence, the slope of the PPF at the corner point continues to become steeper, which means that Foreign cannot completely specialize in manufacturing.

B Why Case 3 switches to Case 2 with the passage of time

As stated above, even if the BGP in Case 3 exists, the economy cannot remain on this path for long. Even if $\bar{\chi}_1 < p$ holds at first, it will become $\bar{\chi}_1 \leq p$ after some time. Then, Case

3 is unsustainable and switches to Case 2. At the very moment when Home switches from Case 3 to Case 2, the absolute value of the rate of change of the terms of trade and that of the MRT are given by

$$|g_p| = (\alpha + \beta)\frac{\dot{K}_1}{K_1} - \alpha[\sigma(t)n_1 + (1-\sigma)n_2], \tag{B-3}$$

$$|g_{\bar{\chi}_1}| = (\alpha + \beta)\frac{\dot{K}_1}{K_1} - \alpha n_1, \tag{B-4}$$

where $\sigma(t) \equiv L_1(t)/[L_1(t) + L_2(t)]$. From these, we obtain

$$|g_p| - |g_{\bar{\chi}_1}| = \alpha(n_1 - n_2)[1 - \sigma(t)] \geq 0. \tag{B-5}$$

When $n_1 > n_2$, $\sigma(t)$ converges to unity after enough time has passed. Then, we have $|g_p| = |\bar{\chi}_1|$, and both p and $\bar{\chi}_1$ decrease at the same rate.

References

Arrow, K. J. (1962) "The economic implications of learning by doing," *Review of Economic Studies* 29, pp. 155–173.

Christiaans, T. (2008) "International trade and industrialization in a non-scale model of economic growth," *Structural Change and Economic Dynamics* 19, pp. 221–236.

Dinopoulos, E. and Thompson, P. (1998) "Schumpeterian growth without scale effects," *Journal of Economic Growth* 3, pp. 313–35.

Felbermayr, G. J. (2007) "Specialization on a technologically stagnant sector need not be bad for growth," *Oxford Economic Papers* 59, pp. 682–701.

Howitt, P. (1999) "Steady endogenous growth with population and R&D inputs growth," *Journal of Political Economy* 107, pp. 715–30.

Jones, C. I. (1995) "R&D-based models of economic growth," *Journal of Political Economy* 103, pp. 759–784.

Jones, C. I. (1999) "Growth: with or without scale effects?" *American Economic Review* 89, pp. 139–44.

Kaneko, A. (2000) "Terms of trade, economic growth, and trade patterns: a small open-economy case," *Journal of International Economics* 52, pp. 169–181.

Kaneko, A. (2003) "The long-run trade pattern in a growing economy," *Journal of Economics* 79 (1), pp. 1–17.

Kortum, S. (1997) "Research, patenting, and technological change," *Econometrica* 65, pp. 1389–419.

Krugman, P. R. (1981) "Trade, accumulation, and uneven development," *Journal of Development Economics* 8, pp. 149–161.

Matsuyama, K. (1992) "Agricultural productivity, comparative advantage, and economic growth," *Journal of Economic Theory* 58, pp. 317–334.

Peretto, P. F. (1998) "Technological change and population growth," *Journal of Economic Growth* 3, pp. 283–311.

Sasaki, H. (2011a) "Trade, non-scale growth, and uneven development," *Metroeconomica* 62 (4), pp. 691–711.

Sasaki, H. (2011b) "Population growth and north-south uneven development," *Oxford Economic Papers* 63, pp. 307–330.

Segerstrom, P. (1998) "Endogenous growth without scale effects," *American Economic Review* 88, pp. 1290–310.

Wong, K.-Y., and Yip, C. K. (1999) "Industrialization, economic growth, and international trade," *Review of International Economics* 7, pp. 522–540.

Young, A. (1998) "Growth without scale effects," *Journal of Political Economy* 106, pp. 41–63.

Figures

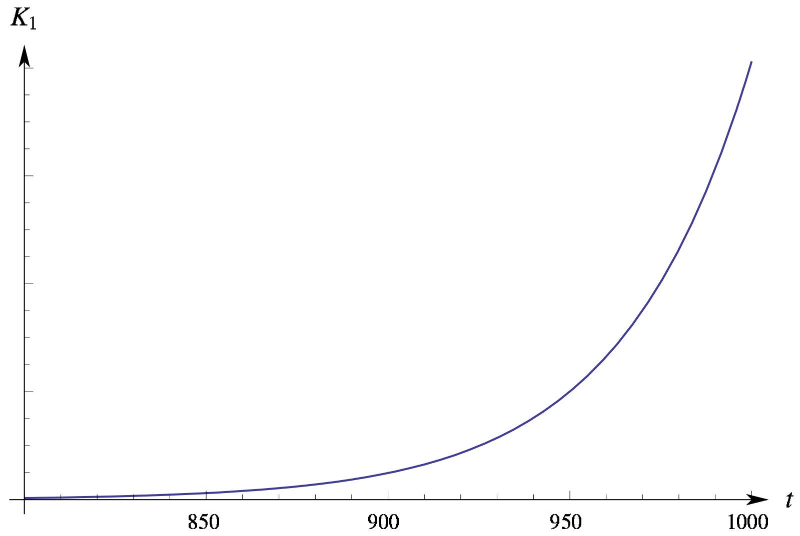

Figure 1: Capital stock in Home in Case 1

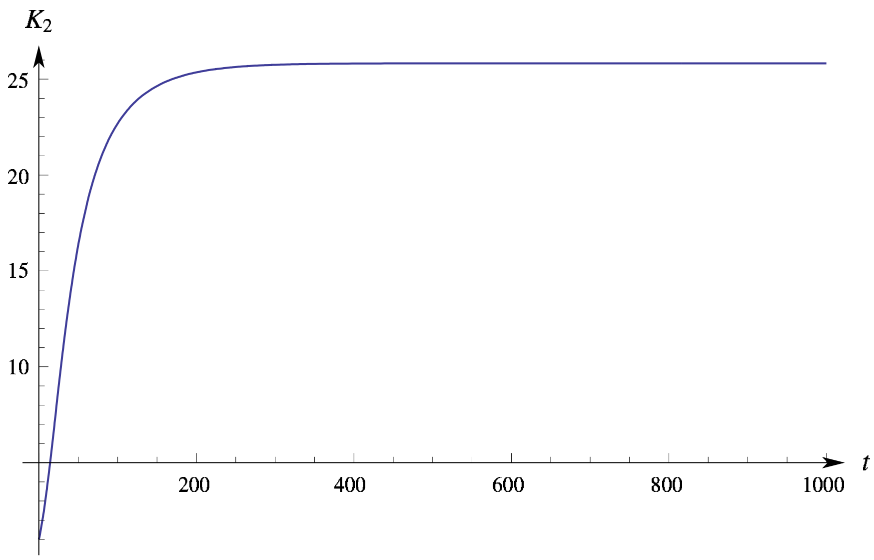

Figure 2: Capital stock in Foreign in Case 1

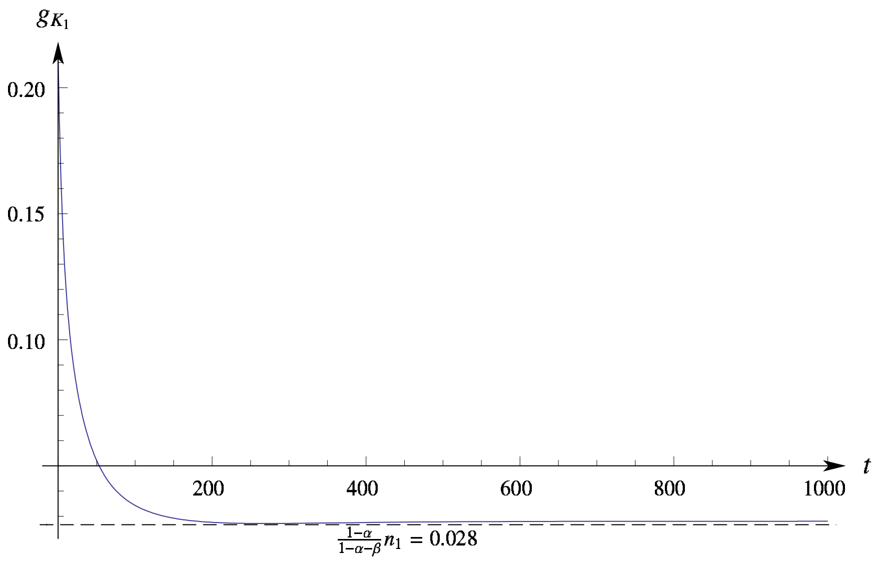

Figure 3: Growth rate of capital stock in Home in Case 1

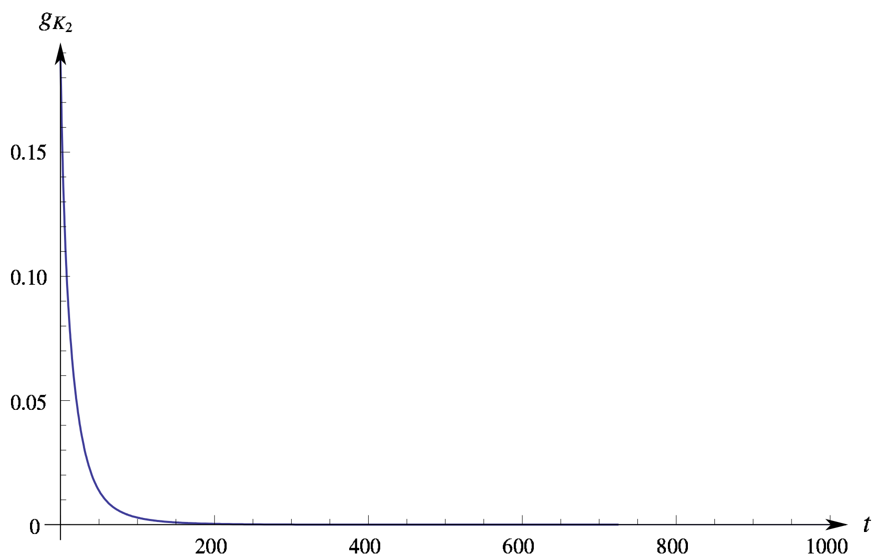

Figure 4: Growth rate of capital stock in Foreign in Case 1

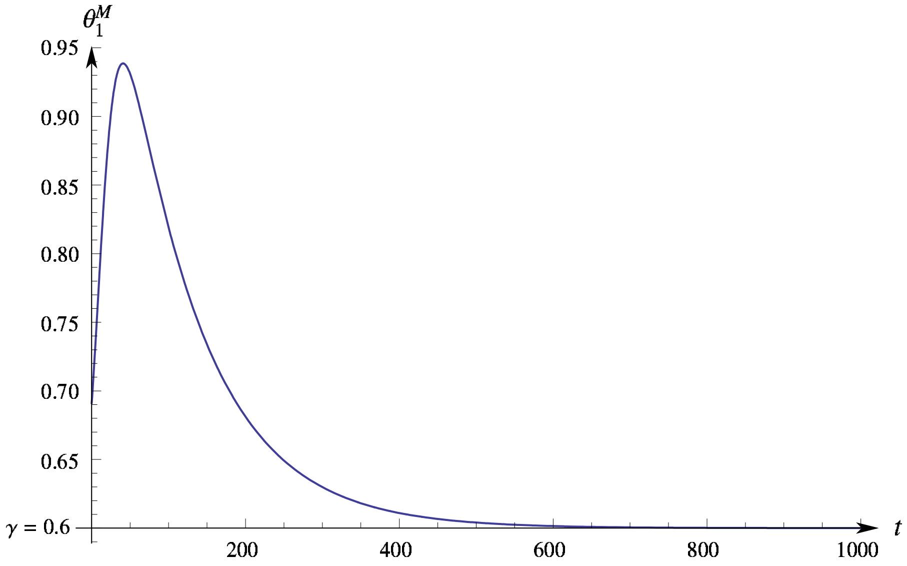

Figure 5: Manufacturing employment share in Home in Case 1

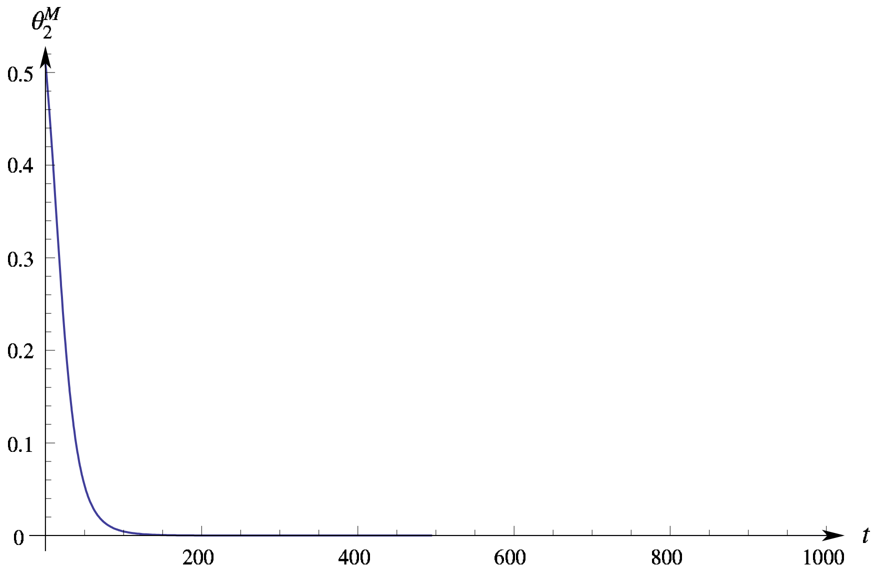

Figure 6: Manufacturing employment share in Foreign in Case 1

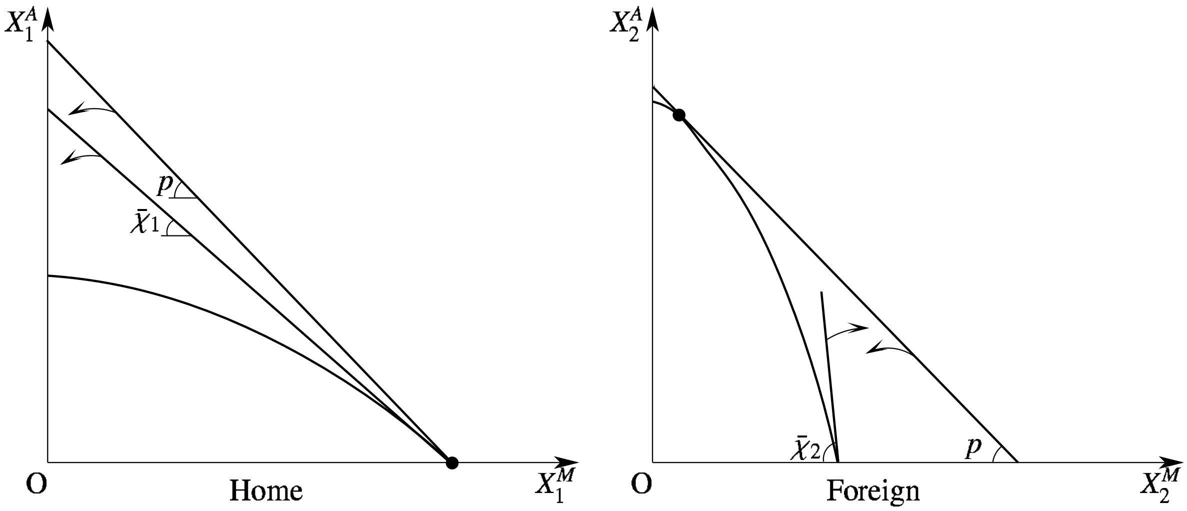

Figure 7: Production possibility frontiers in Home and Foreign

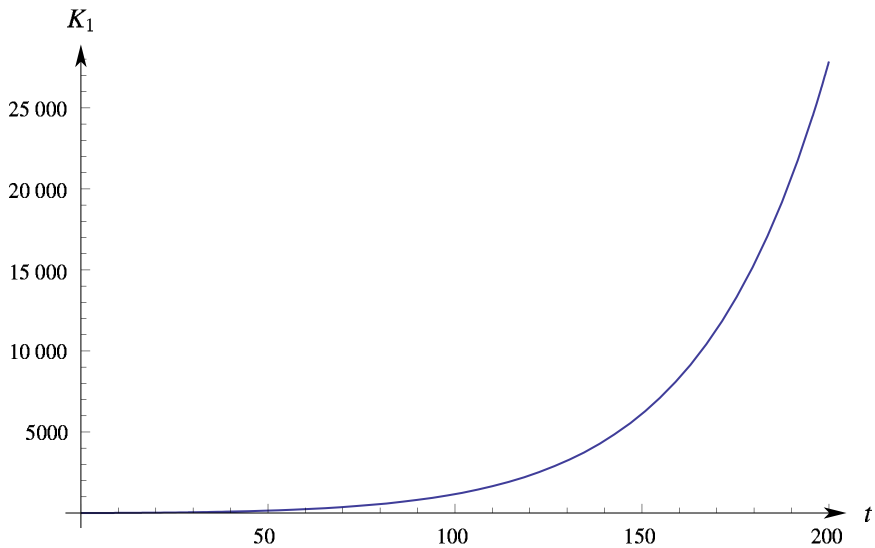

Figure 8: Capital stock in Home in Case 4

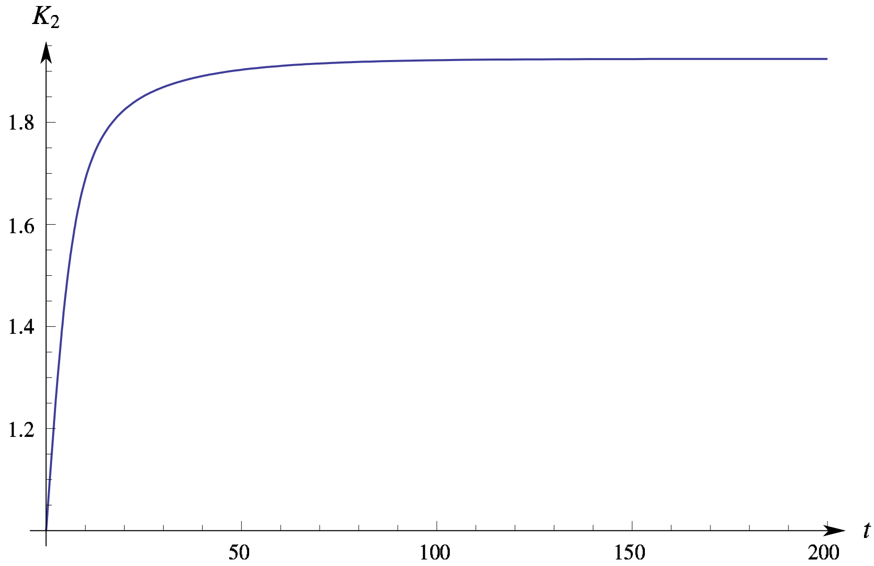

Figure 9: Capital stock in Foreign in Case 4

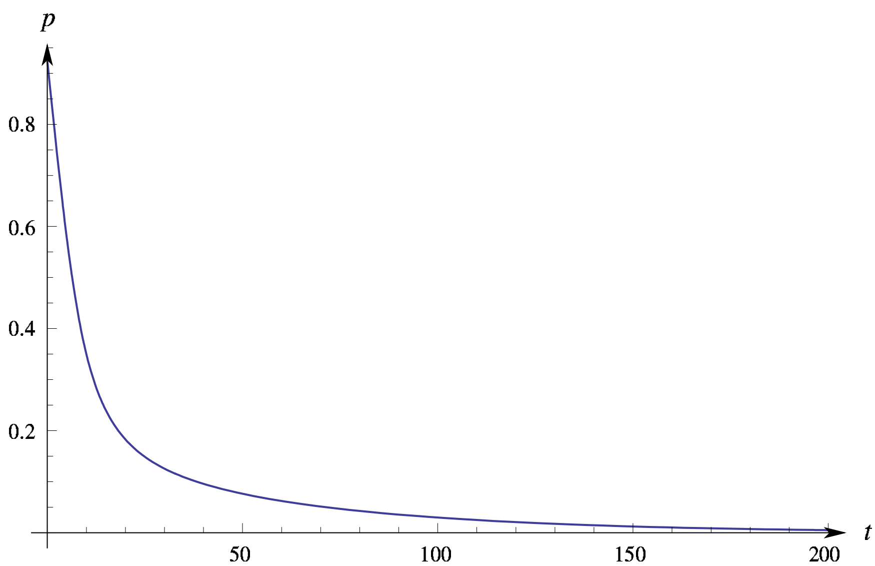

Figure 10: Terms of trade in Case 4

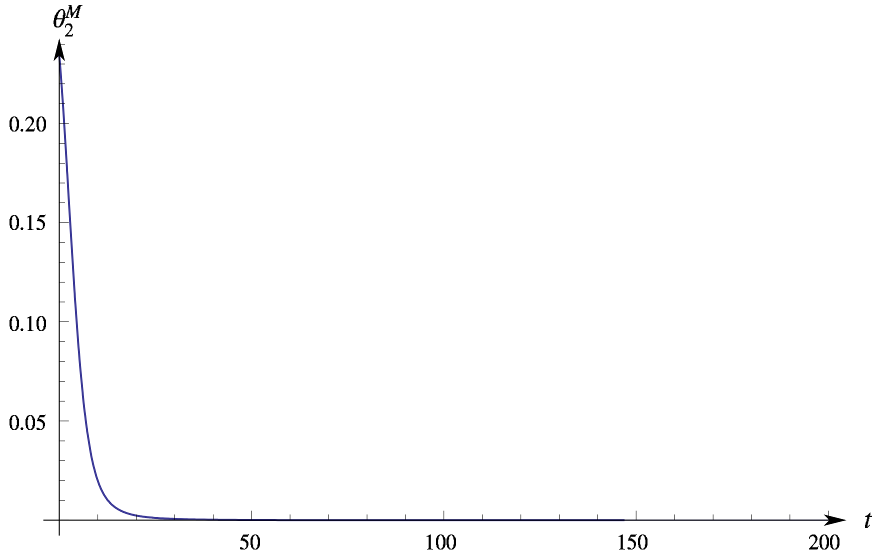

Figure 11: Manufacturing employment share in Foreign in Case 4