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Hiroaki Sasaki and Shinya Fujita

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Research Project Center
Graduate School of Economics
Kyoto University
Yoshida-Hommachi, Sakyo-ku
Kyoto City, 606-8501, Japan

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Income distribution, debt accumulation, and financial fragility in a Kaleckian model with labor supply constraints

Hiroaki SASAKI[†] and Shinya FUJITA[‡]

Abstract

This paper investigates the effect of changes in the retention ratio, profit share, interest rate, and natural rate of growth on the rate of capital accumulation and the financial structure of firms by using a Kaleckian growth model with labor supply constraints. We show that if the economy exhibits a debt-burdened regime, depending on certain conditions, there could be cyclical fluctuations such that the financial structure of firms changes periodically from speculative finance to Ponzi finance.

Keywords: Financial structure, labor supply constraints, Kaleckian model, cyclical fluctuations

JEL Classification: E12; E21; E22; E32; E44

1. Introduction

We introduce labor supply constraints into a Kaleckian model with debt accumulation and investigate the dynamics of the rate of capital accumulation, debt-capital ratio, and capital-labor supply ratio. We further analyze how changes in the labor supply growth, retention ratio of firms, profit share, and interest rate affect the equilibrium values of endogenous variables and the Minskyan financial structure of firms.

In the era of financialization, it is a stylized fact that we observe a tendency of income distribution in favor of shareholders. For example, Stockhammer (2004) points out the increased income share of rentiers in France, Germany, the UK, and the US. Epstein and Power (2003) reveal that in the OECD countries, after the 1980s, the income share of rentiers has been rapidly increasing and the profit share

[†] Graduate School of Economics, Kyoto University

[‡] Graduate School of Economics, Nagoya University

of non-financial corporations mildly increasing. Skott and Ryoo (2008) show that in the US economy, the retention ratio of firms declined from 1970 to 2005.

This tendency also holds true in the Japanese economy. The corporate governance of Japanese firms has turned to shareholder value orientation after the late 1990s, and accordingly the propensity to distribute dividend has increased. In other words, the retention ratio has decreased.[1] The main reason for this is that the old custom of mutual shareholding collapsed and the number of foreign shareholders increased rapidly. As Figure 1 shows, the retention ratio of Japanese firms has consistently exceeded 60% from the late 1960s through the late 1980s, but has been consistently less than 30% from the late 1990s to the present day.[2] These changes in income distribution in Japan are more dramatic than in the US economy, which is regarded as a typical example of financialization.

[Figure 1 around here]

Many empirical and theoretical studies investigate the effect of financialization in terms of income distribution in favor of shareholders on the macro economy.[3] To begin with, we turn to empirical studies.

Stockhammer (2004), which is already cited above, shows that in France, the UK, and the US, the increased income share of rentiers depresses equipment investment of firms. A related study is Orhangazi (2008). He empirically shows that financialization has a negative impact on capital accumulation in the US.

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[1] If we wish to examine the structure of income distribution from the viewpoint of shareholder value orientation, it is appropriate to use the retention ratio that regulates the distribution between shareholders and firms rather than the sum of interest payments and dividends.

[2] The retention ratio is calculated by dividing the retained earnings (which is defined as the net profits of a term − dividend at the term end − remuneration of officials at the term end) by profits (which is defined as the retained earnings + dividend at the term end).

[3] For the microeconomic effects of financialization on firms' behavior, refer to Dallery (2009), in which the effect of financialization on the finance frontier and the expansion frontier are analyzed from various angles.

Increases in financial investment and the opportunity of financial profitability crowd out real investment by changing the incentives of managers. Moreover, increased payments to financial markets depress real investments by decreasing the availability of internal funds. Van Treeck (2008) shows that increases in the profit share and the income share of rentiers can be sources of increased profitability by increasing consumption demand on the one hand and not leading to capital accumulation on the other hand. Onaran, Stockhammer, and Grafl (2011) analyze how financialization and the resulting changes in income distribution affect the aggregate demand in the US economy. They conclude that changes in income distribution in favor of capital have a positive effect on consumption demand though the wealth effect, but has a negative effect on investment demand, and these two effects offset each other and, consequently, in total, have a neutral effect on the aggregate demand.

However, the former two studies (i.e., Stockhammer, 2004; Orhangazi, 2008) estimate only investment functions and do not consider the effect of financialization through consumption demand. Moreover, they are short-run analysis in that they do not consider capital/debt accumulation; in other words, they do not consider endogenous changes in the debt-capital ratio. The latter two studies (i.e., van Treeck, 2008; Onaran, Stockhammer, and Grafl, 2011) consider the effect of financialization on both consumption and investment, and hence are more general than the former two studies. However, these are also short-run analysis in that they too do not consider capital accumulation like the former studies.

Next, we turn to theoretical studies.

Hein and van Treeck (2010a) analytically investigate the effect of increased shareholder power on the macro variables by using a Kaleckian model.[4] They use

[4] In this respect, we consider Hein and van Treeck (2010b) as a related study. They theoretically analyze whether financialization has an expansive effect or contractive effect on the macro economy from some aspects. Using a Kaleckian model, Hein (2010) analytically investigates how an increasing shareholder power can affect the macro economy. He considers the medium term, where the outside finance-capital ratio changes, as well as the short term.

two types of investment functions: the Kalecki-type investment function and Bhaduri and Marglin's (1990) investment function. They show that in accordance with the size of parameters, various demand regimes are produced. Hein (2012) builds a Kaleckian model that considers the effect of a rise in shareholder power on productivity growth, and theoretically investigates its effect on the rate of capital accumulation and growth rate of labor productivity. He concludes that even if the effect of the rising shareholder power is considered, the total effect of financialization on the macro economy is likely to be negative.

These theoretical studies do not consider capital accumulation and hence can be said to be short-run analysis, similar to the above-mentioned empirical studies.

A long-run theoretical analysis that considers capital accumulation is Skott and Ryoo (2008). This study first shows that the US economy experiences financialization, by using some economic indicators. Then, using a Harrodian model and a Kaleckian model (without labor supply constraints), they theoretically investigate the effect of financialization on the macro variables. Limiting our discussion to a Kaleckian model in this paper, we find that the effect of financialization on the macro economy is expansionary.

As stated at the beginning, we investigate the effect of financialization on the macro variables by using a Kaleckian model. In order to carry out a long-run analysis, we incorporate labor supply constraints into a Kaleckian model with debt accumulation. If we intend to investigate the financialization of a capitalist economy in which near-full employment is attained, we need to build a mature economic model in which labor supply constraints bind.

To our knowledge, Ryoo and Skott (2008) is the only study that examines financialization by using a Kaleckian model with labor supply constraints. This study is a continuation of Skott and Ryoo (2008), and theoretically shows that the effect of financialization is expansionary even in a labor-constrained economy. In the present paper, we also build a similar Kaleckian model. However, our method and objective are largely different from those of Ryoo and Skott (2008).

First, Ryoo and Skott (2008) use numerical calculations to conduct their comparative statics analysis. This is because their model is complicated and, hence, analytical solutions are hard to obtain. In this paper, in contrast, we simplify their somehow complicated settings and obtain our main results of

comparative statics analysis by using an analytical method.

Second, their analysis is limited to the steady-state equilibrium. Hence, they do not consider the stability and transitional dynamics of the long-run equilibrium. In this paper, in contrast, we explicitly investigate the stability and transitional dynamics of the long-run equilibrium. According to our analysis, the long-run equilibrium can be stable or unstable, and depending on conditions, there can be perpetual business cycles.

Moreover, we employ the Minskyan taxonomy of the financial structure of firms. That is, we investigate whether the financial structure of firms becomes hedge finance, speculative finance, or Ponzi finance.

As existing studies that investigate the financial structure of firms using Kaleckian models, we consider Foley (2003), Lima and Meirelles (2007), Charles (2008a), and Nishi (2012). In these models, the financial structure of firms is shown to endogenously change along with transition to the steady state, and continues to stay as speculative or Ponzi finance after the economy reaches the steady-state equilibrium.[5]

In contrast, we show that depending on conditions, there can occur cyclical fluctuations such that the financial structure of firms changes periodically from speculative finance and Ponzi finance. Moreover, by using numerical simulations, we show that a decrease in the retention ratio, an increase in the profit share, a decrease in the interest rate, and an increase in the natural rate of growth can diminish cyclical fluctuations.

The rest of the paper is organized as follows. Section 2 derives the dynamical equations of rate of capital accumulation, debt-capital ratio, and capital-effective labor supply ratio. Section 3 obtains the steady-state equilibrium, investigates which financial regime (hedge finance, speculative finance, or Ponzi finance) firms belong to, and in addition examines the stability of the steady-state

[5] A Kaleckian model of Fujita and Sasaki (2011) analytically examines the effect of financialization on macro-economic variables, and investigates how financialization affects the financial structure of firms in the long-run equilibrium by using Minskyan taxonomy. However, they analyze only the long-run equilibrium, but not the transitional dynamics to the long-run equilibrium.

equilibrium. Section 4 investigates the effects of changes in the retention ratio of firms, profit share, interest rate, and the natural rate of growth on the financial structure of firms, rate of capacity utilization, capital-effective labor supply ratio, and rate of employment. Section 5, by using numerical simulations, analyzes how the financial structure of firms evolves along the transitional path to the steady state. Section 6 concludes the paper.

2. Model

We consider a one-sector, one-good, closed economy, with no government sector. In addition, there are three types of agents—the firms, households, and banks. The firms produce using the capital stock and labor services supplied by the households. According to the post-Keynesian Horizontalist approach (Moore, 1988; Rochon, 1999), we assume that firms borrow their investment funds from households via the banking sector at a constant nominal lending interest rate. The firms then engage in investments using the borrowings and their retained earnings. In addition, they do not issue new equity for investment, but issue equity at the time when they start a business. These equities are all held by the households. Therefore, the households receive wage income, interest income, and dividend income (Taylor, 2004; Skott and Ryoo, 2008; Ryoo and Skott, 2008). The households spend a constant ratio of their total income on consumption and save the rest. The banks lend the deposits they receive from the households to the firms and, in addition, set a nominal lending interest rate. However, we assume that the deposit interest rates and the lending interest rates are equal, and hence the banks incur no cost and make no profit.

2.1 Dynamics of the rate of capital accumulation

Manufacturing firms produce with a fixed coefficient production function. In this paper, we assume that the potential output-capital ratio is constant. If we denote the actual output as Y, then we can denote the rate of capacity utilization as $u = Y/K$.[6] Firms obtain profits, which are obtained by subtracting the dividend

[6] Strictly speaking, the rate of capacity utilization in this paper is the output-capital ratio. The rate of capacity utilization is decomposed into the output-capital ratio and the

and interest payments with regard to their real debt L, and so the profits of firms (i.e., the retained earnings) divided by K, that is, Π_f, are obtained by

$$\frac{\Pi_f}{K} = s_f\left(\pi u - i\frac{L}{K}\right), \quad s_f \in (0,1), \quad \pi \in (0,1), \quad i > 0, \tag{1}$$

where s_f denotes the retention ratio, π the profit share, and i the nominal interest rate, all assumed to be constant.

The households in the economy receive wage income, interest income, and dividend income, and so their total income divided by K, that is, Π_h, is obtained by

$$\frac{\Pi_h}{K} = (1-\pi)u + i\frac{L}{K} + (1-s_f)\left(\pi u - i\frac{L}{K}\right). \tag{2}$$

Let a constant s_h be the households' propensity to save. From equations (1) and (2), the actual rate of capital accumulation (i.e., the real saving divided by capital stock) is obtained by

$$g \equiv \frac{S}{K} = s_f(\pi u - i\lambda) + s_h\left[(1-s_f\pi)u + s_f i\lambda\right], \quad s_h \in (0,1), \tag{3}$$

where $\lambda = L/K$ denotes the debt-capital ratio.

From equation (3), we obtain the relationship between the rate of capacity utilization and the rate of capital accumulation:

$$u = \frac{g + s_f(1-s_h)i\lambda}{s_f\pi(1-s_h) + s_h}. \tag{4}$$

We assume that the firms' target rate of capital accumulation g^d is an increasing function of the profit rate and capacity utilization rate (Kalecki, 1954; Taylor, 2004; Hein, 2006) and a decreasing function of the employment rate e (Ryoo and Skott, 2008; Abe, 2012; Skott and Zipperer, 2012).

$$g^d = \alpha + \beta s_f(\pi u - i\lambda) + \gamma u - \delta e, \quad \alpha > 0, \quad \beta > 0, \quad \gamma > 0, \quad \delta > 0. \tag{5}$$

The negative effect of the employment rate on investment is justified by the following two factors. First, as the employment rate increases and the economy approaches full employment, the bargaining power of workers increases while the

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capital-potential output ratio. If we assume that the capital-potential output ratio is constant, then the rate of capacity utilization and the output-capital ratio change in the same direction. Therefore, we can regard the output-capital ratio as the rate of capacity utilization.

firms' animal spirit decreases. Second, if the economy is near full employment, the firms' opportunity to employ more workers and expand production is lost, and their incentive to accumulate capital stock is also lost (Ryoo and Skott, 2008).

Next, if we denote employment as E, consider $A = Y/E$ the level of labor productivity, and in addition denote the labor supply as N, the employment rate can be shown as follows:

$$e = \frac{E}{N} = \frac{Y}{K} \cdot \frac{K}{AN} = u\tilde{k} \tag{6}$$

where $\tilde{k} = K/(AN)$ denotes the capital-effective labor supply ratio. Substituting equation (6) into equation (5), we obtain

$$g^d = \alpha + \beta s_f (\pi u - i\lambda) + (\gamma - \delta\tilde{k})u . \tag{5'}$$

We assume that the actual rate of capital accumulation is adjusted according to the difference between the target rate of capital accumulation and the actual rate of capital accumulation (Skott and Zipperer, 2012):

$$\dot{g} = \phi(g^d - g), \quad \phi > 0 , \tag{7}$$

where the dot over the variable denotes the time derivative of the variable, and ϕ the speed of adjustment. Substituting equations (4) and (5') into equation (7), we obtain

$$\dot{g} = g(g, \lambda, \tilde{k}) \equiv \phi\left\{ -\frac{s_f \pi(1 - s_h - \beta) + s_h - \gamma + \delta\tilde{k}}{s_f \pi(1 - s_h) + s_h} g + \frac{\left[(\gamma - \delta\tilde{k})(1 - s_h) - \beta s_h\right]}{s_f \pi(1 - s_h) + s_h} s_f i\lambda + \alpha \right\} . \tag{8}$$

2.2 Dynamics of the debt-capital ratio

Log-differentiating the debt-capital ratio, we obtain

$$\dot{\lambda} = \left(\frac{\dot{L}}{L} - \frac{\dot{K}}{K} \right) \lambda = \frac{\dot{L}}{K} - g\lambda . \tag{9}$$

The increment of debt is equal to the actual investment less the retained earnings. Therefore, with equation (4), the increment of debt can be given as follows:

$$\frac{\dot{L}}{K} = g - s_f(\pi u - i\lambda) = \frac{s_h(1 - s_f \pi)}{s_f \pi(1 - s_h) + s_h} g + \frac{s_f s_h}{s_f \pi(1 - s_h) + s_h} i\lambda . \tag{10}$$

Substituting equation (10) into equation (9), we obtain the dynamical equation of the debt-capital ratio:

$$\dot{\lambda} = \lambda(g,\lambda) \equiv \frac{s_h(1-s_f\pi)-\left[s_f\pi(1-s_h)+s_h\right]\lambda}{s_f\pi(1-s_h)+s_h} g + \frac{s_f s_h}{s_f\pi(1-s_h)+s_h} i\lambda . \tag{11}$$

2.3 Dynamics of the capital-effective labor supply ratio

We assume that the productivity and supply of labor grow at constant rates n_1 and n_2, respectively. Log-differentiating the capital-effective labor supply ratio, we obtain the dynamical equation of $\tilde{k}$,

$$\dot{\tilde{k}} = \tilde{k}(g) \equiv (g-n)\tilde{k} , \quad n>0 \tag{12}$$

where $n \equiv n_1 + n_2$, which we call the "natural rate of growth."

3. Properties of the steady-state equilibrium

3.1 Steady-state values

From the above analysis, the dynamical system of the rate of capital accumulation, debt-capital ratio, and capital-effective labor supply ratio can be given as follows:

$$\dot{g} = g\left(g,\lambda,\tilde{k}\right) \equiv \phi\left\{-\frac{s_f\pi(1-s_h-\beta)+s_h-\gamma+\delta\tilde{k}}{s_f\pi(1-s_h)+s_h} g + \frac{\left[\left(\gamma-\delta\tilde{k}\right)(1-s_h)-\beta s_h\right]}{s_f\pi(1-s_h)+s_h} s_f i\lambda + \alpha\right\}, \tag{8}$$

$$\dot{\lambda} = \lambda(g,\lambda) \equiv \frac{s_h(1-s_f\pi)-\left[s_f\pi(1-s_h)+s_h\right]\lambda}{s_f\pi(1-s_h)+s_h} g + \frac{s_f s_h}{s_f\pi(1-s_h)+s_h} i\lambda , \tag{11}$$

$$\dot{\tilde{k}} = \tilde{k}(g) \equiv (g-n)\tilde{k} . \tag{12}$$

The steady-state equilibrium is a situation where $\dot{g} = \dot{\lambda} = \dot{\tilde{k}} = 0$. Let the steady-state values be denoted as g^*, λ^*, and $\tilde{k}^*$.

To begin with, equation (12) indicates that the steady-state value of the rate of capital accumulation is equal to the natural rate of growth:[7]

$$g^* = n \tag{13}$$

Next, substituting equation (13) into equation (11), we obtain the steady-state value of the debt-capital ratio as follows:

$$\lambda^* = \frac{s_h n(1-s_f\pi)}{\left[s_f\pi(1-s_h)+s_h\right]n - s_f s_h i} . \tag{14}$$

If the interest rate is sufficiently small, then $\left[s_f\pi(1-s_h)+s_h\right]n - s_f s_h i > 0$ is satisfied, implying that the steady-state value of the debt-capital ratio is positive.

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[7] We here exclude trivial solution $\tilde{k}^* = 0$.

Moreover, equation (14) can be rewritten as follows:

$$\lambda^* = \frac{s_h n(1 - s_f \pi)}{s_h n(1 - s_f \pi) + s_f (\pi n - s_h i)}. \tag{14'}$$

Note here that $\lambda^* < 1$ holds under $\pi n - s_h i > 0$, $\lambda^* = 1$ holds under $\pi n - s_h i = 0$, and $\lambda^* > 1$ holds under $\pi n - s_h i < 0$. The situation where the steady-state value of the debt-capital ratio is greater than or equal to unity seems to be unrealistic. Thus, it is appropriate to assume that $\pi n - s_h i > 0$ in the following analysis. Under this assumption, $[s_f \pi(1 - s_h) + s_h]n - s_f s_h i > 0$ is also satisfied. Furthermore, equation (14) can be rewritten as follows:

$$n = \frac{s_f s_h i \lambda^*}{[s_f \pi(1 - s_h) + s_h]\lambda^* - s_h(1 - s_f \pi)} > 0. \tag{15}$$

The denominator of the right-hand side of equation (15) is positive. This is because substituting equation (14) into $[s_f \pi(1 - s_h) + s_h]\lambda^* - s_h(1 - s_f \pi) > 0$ yields $s_f s_h^2 i(1 - s_f \pi) > 0$.

Finally, substituting equations (13) and (14) into equation (8) yields the steady-state value of the capital-effective labor supply ratio as follows:

$$\tilde{k}^* = \frac{[s_f \pi(1 - s_h) + s_h]\alpha - [s_f \pi(1 - s_h - \beta) + s_h - \gamma]g^* + s_f[\gamma(1 - s_h) - \beta s_h]i\lambda^*}{\delta[g^* + s_f(1 - s_h)i\lambda^*]}. \tag{16}$$

The denominator of the right-hand side of equation (16) is positive, whereas the numerator is ambiguous. If the trend term in the target rate capital accumulation, α, is sufficiently large, then the steady-state value of the capital-effective labor supply ratio becomes positive.

From the above analysis, the following assumptions are required in order to obtain economically meaningful steady-state values.

Assumption 1. $\pi n - s_h i > 0$ holds.

Assumption 2. $[s_f \pi(1 - s_h) + s_h]\alpha - [s_f \pi(1 - s_h - \beta) + s_h - \gamma]g^* + s_f[\gamma(1 - s_h) - \beta s_h]i\lambda^* > 0$ holds.

3.2 Financial structure of firms

Minsky (1975, 1982) classified the financial structure of firms into three regimes

based on cash-flow accounting: hedge finance, speculative finance, and Ponzi finance. Our next task is to find out the financial regime the steady-state financial structure locates in. Earlier studies that specify the taxonomy of the financial structure using Kaleckian models include Foley (2003, pp.158–160), Meirelles and Lima (2006, pp.102–104), Lima and Meirelles (2007, pp.570–572), Charles (2008a, pp.320–321), Fujita and Sasaki (2011, pp.8–10), and Nishi (2012, pp.12–14).[8]

[Table 1 around here]

Table 1 shows the condition for each financial regime, based on the present notation. Hedge finance is a situation where the financial structure is the most sound; it is defined as a situation where the profits of firms are larger than or equal to their total expenditure, that is, the sum of their investment, interest payments, and dividends. Rearranging the condition for hedge finance obtains the following relation:

$$g - s_f\left(\pi u - i\lambda\right) \leq 0 . \tag{17}$$

Speculative finance is defined as a situation where the profits of firms are less than the sum of their investment, interest payments, and dividends but larger than the sum of their interest payments and dividends. Rearranging the conditions for speculative finance obtains the following relations:

$$g - s_f\left(\pi u - i\lambda\right) > 0 \quad \text{and} \quad \pi u > i\lambda . \tag{18}$$

Ponzi finance is a situation where the finance structure is the most fragile; it is defined as a situation where the profits of firms are less than the sum of their interest payments and dividends. Rearranging the condition for Ponzi finance obtains the following relation:

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[8] These earlier studies include only investment and interest payments in firms' expenditure; they do not consider dividends. However, we should not abstract from dividends, because it is one of the most important expenditure items under financialization. In addition, Minsky himself explicitly explains that dividend is a component of the firms' expenditure. "The cash payments made by a unit over a relevant time period equal the spending on current labor and purchased inputs, tax payments, the remittance due to debts that fall due, and dividends." (Minsky, 1982, p.24)

$$\pi u \le i\lambda . \tag{19}$$

In which financial regime is the steady-state equilibrium located? First, by substituting equation (4) into equation (17), we obtain the following inequality:

$$\left(1 - s_f \pi\right) g + s_f i\lambda \le 0 . \tag{20}$$

Equation (20) contradicts the fact that both the rate of capital accumulation and debt-capital ratio show positive signs, which implies that the steady-state equilibrium is not hedge finance. In addition, this means that the former condition of equation (18) necessarily holds.

Second, substituting equation (4) into the latter inequality of equation (18) and rearranging, we obtain the following relation:

$$\pi g - s_h i\lambda > 0 . \tag{21}$$

If equation (21) holds, the financial structure of firms becomes a speculative regime; otherwise, it becomes a Ponzi finance regime. Substituting equations (13) and (14) into equation (21) produces the following inequality:

$$\pi n - s_h i > 0 . \tag{22}$$

Equation (22) identifies with Assumption 1. Thus, the financial position of firms in the steady-state equilibrium necessarily becomes speculative finance when the steady-state value of the debt-capital ratio is smaller than unity. Moreover, if $\pi n - s_h i < 0$ holds (i.e., the financial structure is in the Ponzi finance regime), the steady-state value of the debt-capital ratio is larger than unity, and if $\pi n - s_h i = 0$ holds (i.e., the financial structure is in a boundary between speculative and Ponzi finance regimes), the steady-state value is equal to unity.

3.3 Stability

We next consider the local stability condition of the steady-state equilibrium. The elements of the Jacobian matrix **J** that correspond to the system of the differential equations are given as follows:

$$J_{11} \equiv \frac{\partial \dot{g}}{\partial g} = -\phi \frac{s_f \pi\left(1 - s_h - \beta\right) + s_h - \gamma + \delta\tilde{k}^*}{s_f \pi\left(1 - s_h\right) + s_h} < 0 , \tag{23}$$

$$J_{12} \equiv \frac{\partial \dot{g}}{\partial \lambda} = \phi \frac{s_f \left[\left(\gamma - \delta\tilde{k}^*\right)\left(1 - s_h\right) - \beta s_h\right] i}{s_f \pi\left(1 - s_h\right) + s_h} , \tag{24}$$

$$J_{13} \equiv \frac{\partial \dot{g}}{\partial \tilde{k}} = -\phi\delta \frac{g^* + s_f(1-s_h)i\lambda^*}{s_f\pi(1-s_h)+s_h} < 0, \tag{25}$$

$$J_{21} \equiv \frac{\partial \dot{\lambda}}{\partial g} = \frac{s_h(1-s_f\pi)-\left[s_f\pi(1-s_h)+s_h\right]\lambda^*}{s_f\pi(1-s_h)+s_h} < 0, \tag{26}$$

$$J_{22} \equiv \frac{\partial \dot{\lambda}}{\partial \lambda} = -\frac{\left[s_f\pi(1-s_h)+s_h\right]g^* - s_f s_h i}{s_f\pi(1-s_h)+s_h} < 0, \tag{27}$$

$$J_{23} \equiv \frac{\partial \dot{\lambda}}{\partial \tilde{k}} = 0, \tag{28}$$

$$J_{31} \equiv \frac{\partial \dot{\tilde{k}}}{\partial g} = \tilde{k}^* > 0, \tag{29}$$

$$J_{32} \equiv \frac{\partial \dot{\tilde{k}}}{\partial \lambda} = 0, \tag{30}$$

$$J_{33} \equiv \frac{\partial \dot{\tilde{k}}}{\partial \tilde{k}} = 0. \tag{31}$$

Note that all the elements are evaluated at the steady-state values.

First, we suppose the situation where quantity adjustment in the goods market is stable—that is, the Keynesian stability condition is always satisfied. In this situation, we have $J_{11} < 0$.

Second, we introduce the following definition.

Definition 1. We define $\left(\gamma - \delta\tilde{k}^*\right)(1-s_h) - \beta s_h > 0$ as the economy where the rate of capital accumulation is debt led, and $\left(\gamma - \delta\tilde{k}^*\right)(1-s_h) - \beta s_h < 0$ as the economy where the rate of capital accumulation is debt burdened.[9]

The element J_{12} shows the effect of an increase in the debt-capital ratio on the rate of capital accumulation. If its sign is positive, the economy in the

[9] In our model, the regime of rate of capital accumulation depends on the level of the endogenous variable, $\tilde{k}$; this differs from earlier researches (Lavoie, 1995; Taylor, 2004; Hein, 2007; Sasaki and Fujita, 2012). For example, if the steady-state value of the capital-effective labor supply ratio is sufficiently large, the rate of capital accumulation in the steady state exhibits a debt-burdened regime, and vice versa.

steady-state equilibrium is debt led, and if the sign is negative, the economy is debt burdened.

Finally, $J_{13} < 0$, $J_{21} < 0$, and $J_{22} < 0$ are obtained under Assumption 1.

The characteristic equation that corresponds to the Jacobian matrix $\mathbf{J}$ is given as

$$q^3 + a_1 q^2 + a_2 q + a_3 = 0, \tag{32}$$

where q denotes a characteristic root. The coefficients of equation (32) are given by

$$a_1 = -\mathrm{tr}\mathbf{J} = -J_{11} - J_{22} > 0, \tag{33}$$

$$a_2 = J_{11}J_{22} - J_{12}J_{21} - J_{13}J_{31}, \tag{34}$$

$$a_3 = -\det\mathbf{J} = J_{13}J_{22}J_{31} > 0, \tag{35}$$

where $\mathrm{tr}\mathbf{J}$ denotes the trace of $\mathbf{J}$ and $\det\mathbf{J}$ denotes the determinant of $\mathbf{J}$.

The necessary and sufficient condition for local stability of the steady-state equilibrium is given by $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$. We clearly know that $a_1 > 0$ and $a_3 > 0$, whereas we do not know the signs of a_2 and $a_1 a_2 - a_3$. In what follows, we investigate whether or not the latter two coefficients are positive.

If the rate of capital accumulation is debt led, we have $J_{12} > 0$, and accordingly, $a_2 > 0$. However, if the rate of capital accumulation is debt burdened, $J_{12} > 0$ is obtained, and hence the sign of a_2 is ambiguous. We therefore assume the following.

Assumption 3. Even if the rate of capital accumulation exhibits a debt-burdened regime, $a_2 > 0$ holds.

This assumption is satisfied in the situation where the trend term α of the target rate of capital accumulation is sufficiently large.[10]

We next examine the sign of $a_1 a_2 - a_3$. The coefficients of the characteristic equation can be rewritten as follows:

$$a_1 = \Delta_1 \phi + \Delta_2, \quad \Delta_1 \equiv -\frac{J_{11}}{\phi} > 0, \quad \Delta_2 \equiv -J_{22} > 0, \tag{33'}$$

[10] See the appendix.

$$a_2 = \Delta_3 \phi, \quad \Delta_3 \equiv \frac{J_{11}J_{22} - J_{12}J_{21} - J_{13}J_{31}}{\phi} > 0, \tag{34'}$$

$$a_3 = \Delta_4 \phi, \quad \Delta_4 \equiv \frac{J_{13}J_{22}J_{31}}{\phi} > 0, \tag{35'}$$

$$a_1 a_2 - a_3 = f(\phi) \equiv \phi(\Delta_1 \Delta_3 \phi + \Delta_2 \Delta_3 - \Delta_4). \tag{36}$$

Note here that Δ_1, Δ_2, Δ_3, and Δ_4 are all positive and independent of the level of ϕ. From this, $f(\phi)$ shows a parabola, with its vertex oriented downward in the $(\phi, a_1 a_2 - a_3)$ plane. Furthermore, this quadratic function passes the origin and $((\Delta_4 - \Delta_2\Delta_3)/\Delta_1\Delta_3, 0)$. Now, we rewrite $\Delta_4 - \Delta_2\Delta_3$ as follows:

$$\Delta_4 - \Delta_2\Delta_3 = \frac{J_{22}(J_{11}J_{22} - J_{12}J_{21})}{\phi}. \tag{37}$$

If the rate of capital accumulation exhibits a debt-led regime, $\Delta_4 - \Delta_2\Delta_3 < 0$ is obtained, because $J_{12} > 0$. Then, $f(\phi) > 0$ is always satisfied for the range of $\phi > 0$. Thus, the steady-state equilibrium is locally stable in the debt-led case, because we have $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$.

However, if the rate of capital accumulation exhibits a debt-burdened regime, $J_{11}J_{22} - J_{12}J_{21}$ shows an ambiguous sign, because $J_{12} < 0$.[11] In the case of $J_{11}J_{22} - J_{12}J_{21} > 0$, we have $\Delta_4 - \Delta_2\Delta_3 < 0$, and the steady-state equilibrium becomes stable, as was in the debt-led case.

In contrast, in the case of $J_{11}J_{22} - J_{12}J_{21} < 0$, we have $\Delta_4 - \Delta_2\Delta_3 > 0$.[12] Then, for $\phi \in (0, (\Delta_4 - \Delta_2\Delta_3)/\Delta_1\Delta_3)$, we obtain $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $f(\phi) = a_1 a_2 - a_3 < 0$, which implies that the steady-state equilibrium becomes locally unstable. On the other hand, for the range of $\phi > (\Delta_4 - \Delta_2\Delta_3)/\Delta_1\Delta_3$, we obtain $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $f(\phi) = a_1 a_2 - a_3 > 0$, which implies that the steady-state equilibrium becomes locally stable. Finally, the Hopf bifurcation occurs at $\phi = (\Delta_4 - \Delta_2\Delta_3)/\Delta_1\Delta_3$ because $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and

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[11] We cannot prove the determinant factors of the sign of $J_{11}J_{22} - J_{12}J_{21}$ analytically. The numerical simulation in section 5.2, however, indicates that when the rate of capital accumulation is debt burdened, a decrease in the retention ratio, an increase in the profit share, a decline in the interest rate, and a rise in the natural growth rate raise the value of $J_{11}J_{22} - J_{12}J_{21}$ and contribute to the local stability of the steady-state equilibrium.

[12] Note that $J_{11}J_{22} - J_{12}J_{21} < 0$ is not necessarily consistent with Assumption 3.

$f(\phi) = a_1 a_2 - a_3 = 0$ are satisfied. This means that there exists a continuous family of non-constant, periodic solutions of the system around $\phi = (\Delta_4 - \Delta_2\Delta_3)/\Delta_1\Delta_3$.

Summarizing these results, we obtain the following propositions.

Proposition 1. *Suppose that in the steady-state equilibrium the rate of capital accumulation is debt led. Then, the steady-state equilibrium is locally stable.*

Proposition 2. *Suppose that in the steady-state equilibrium the rate of capital accumulation is debt burdened. Then, in the case of* $J_{11}J_{22} - J_{12}J_{21} > 0$*, the steady-state equilibrium is locally stable. In the case of* $J_{11}J_{22} - J_{12}J_{21} < 0$*, a higher adjustment speed of the rate of capital accumulation makes the steady-state equilibrium stable, and a lower adjustment speed of the rate of capital accumulation makes the steady-state equilibrium unstable. Moreover, a limit cycle occurs when the adjustment speed of the rate of capital accumulation approaches near some value.*

4 Comparative statics analysis

4.1 The debt-capital ratio

As section 3.2 has shown, the threshold between speculative finance and Ponzi finance in the steady-state equilibrium is a situation where the debt-capital ratio becomes unity. Accordingly, if the steady-state value of the debt-capital ratio increases and approaches unity, then the financial structure of firms becomes more fragile. On the other hand, if the steady-state value of the debt-capital ratio decreases, then the financial structure of firms becomes more healthy.

The effects of changes in the retention ratio, profit share, interest rate, and natural rate of growth on the steady-state value of the debt-capital ratio are given as follows:

$$\frac{d\lambda^*}{ds_f} = -\frac{s_h n(\pi n - s_h i)}{\{[s_f \pi(1-s_h) + s_h]n - s_f s_h i\}^2} < 0 \tag{38}$$

$$\frac{d\lambda^*}{d\pi} = -\frac{s_f s_h n(n - s_f s_h i)}{\{[s_f \pi(1-s_h) + s_h]n - s_f s_h i\}^2} < 0 \tag{39}$$

$$\frac{d\lambda^*}{di} = \frac{s_f s_h^2 n(1 - s_f \pi)}{\{[s_f \pi(1-s_h) + s_h]n - s_f s_h i\}^2} > 0 \tag{40}$$

$$\frac{d\lambda^*}{dn} = -\frac{s_f s_h^2 i\left(1 - s_f \pi\right)}{\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\}^2} < 0 \tag{41}$$

Note that $n - s_f s_h i > 0$ holds under Assumption 1. Thus, increases in the retention ratio and profit share decrease the debt-capital ratio, because increases in the retention ratio and profit share lower the ratio of borrowings to investment.

In addition, an increase in the interest rate raises the increasing speed of debt accumulation, and hence increases the debt-capital ratio.

Furthermore, since an increase in the natural rate of growth is equivalent to an increase in the rate of capital accumulation, it lowers the debt-capital ratio.

4.2 The rate of capacity utilization

Substituting equations (13) and (14) into equation (4), we obtain the steady-state value of the rate of capacity utilization:

$$u^* = \frac{n\left(n - s_f s_h i\right)}{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i}. \tag{42}$$

From Assumption 1, we have $u^* > 0$.

The effects of changes in the retention ratio, profit share, interest rate, and natural rate of growth on the steady-state value of the rate of capacity utilization are given as follows:

$$\frac{du^*}{ds_f} = -\frac{n^2\left(1 - s_h\right)\left(\pi n - s_h i\right)}{\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\}^2} < 0, \tag{43}$$

$$\frac{du^*}{d\pi} = -\frac{s_f n^2\left(1 - s_h\right)\left(n - s_f s_h i\right)}{\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\}^2} < 0, \tag{44}$$

$$\frac{du^*}{di} = \frac{s_f s_h n^2\left(1 - s_h\right)\left(1 - s_f \pi\right)}{\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\}^2} > 0, \tag{45}$$

$$\frac{du^*}{dn} = \frac{n\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\} - s_f s_h i\left(n - s_f s_h i\right)}{\left\{\left[s_f \pi\left(1 - s_h\right) + s_h\right] n - s_f s_h i\right\}^2}. \tag{46}$$

Equation (43) shows that an increase in the retention ratio decreases the rate of capacity utilization, while equation (44) shows that an increase in the profit share decreases the rate of capacity utilization. In other words, in an economy with labor supply constraints, the steady-state equilibrium exhibits the stagnationist regime (Blecker, 2002). The reasons for obtaining these results are as follows. An

increase in the retention ratio depresses the income of households through a decrease in dividends on the one hand and a decrease in the debt-capital ratio and the resultant decrease in the interest income on the other hand. An increase in the profit share lowers the income of households through a decrease in wage income. These decreases in the income of households depress consumption demand, which results in a decrease in the rate of capacity utilization.[13]

Equation (45) shows that a decrease in the interest rate lowers the rate of capacity utilization. In other words, the rate of capacity utilization exhibits a debt-led regime in the steady state (Lavoie, 1995; Hein, 2007). As stated above, this is also because a decrease in the interest rate lowers the income of households. Note that the results of equations (43) to (45) are the same as those in Ryoo and Skott (2008), which are obtained by numerical calculations.[14]

Finally, equation (46) shows the effect of an increase in the natural rate of growth on the rate of capacity utilization, but its sign is ambiguous.

4.3 The capital-effective labor supply ratio

We now examine the effects of changes in the retention ratio, profit share, interest rate, and natural rate of growth on the capital-effective labor supply ratio.

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[13] Here, one might consider increases in the retention ratio and profit share offsetting decreases in consumption demand by increasing the invest demand through an increase in profits of firms. However, since labor supply constraints bind in the long run, the rate of capital accumulation is equal to the natural rate of growth given exogenously. Hence, the effects of increases in the retention ratio and profit share are entirely reflected on a decrease in the rate of capacity utilization as a decrease in consumption demand. In contrast, in a dual economy where labor supply constraints do not bind, and hence the rate of capital accumulation is not equal to the natural rate of growth, it is possible that an increase in the retention ratio raises the rate of capacity utilization.

[14] However, there are some differences between Ryoo and Skott (2008) and our model with regard to the specifications of the investment and consumption functions. In addition, while new issues of equity are not considered in our model, it is considered in Ryoo and Skott (2008). These facts imply that the results in Ryoo and Skott (2008) can be obtained without using numerical simulations if somehow the complicated settings are ignored.

$$\frac{d\tilde{k}^*}{ds_f} = \frac{j_{31}\left(A_2 J_{12} - A_1 J_{22}\right)}{\det \mathbf{J}}, \tag{47}$$

$$\frac{d\tilde{k}^*}{d\pi} = \frac{j_{31}\left(B_2 J_{12} - B_1 J_{22}\right)}{\det \mathbf{J}}, \tag{48}$$

$$\frac{d\tilde{k}^*}{di} = \frac{j_{31}\left(C_2 J_{12} - C_1 J_{22}\right)}{\det \mathbf{J}}, \tag{49}$$

$$\frac{d\tilde{k}^*}{dn} = \frac{\left(J_{11} J_{22} - J_{12} J_{21}\right)\tilde{k}^*}{\det \mathbf{J}}, \tag{50}$$

where

$$A_1 = \frac{\phi\left[\left(\gamma - \delta\tilde{k}^*\right)\left(1 - s_h\right) - \beta s_h\right]\left(\pi n - s_h i\lambda^*\right)}{\left[s_f \pi\left(1 - s_h\right) + s_h\right]^2}, \quad A_2 = \frac{s_h\left(\pi n - s_h i\lambda^*\right)}{\left[s_f \pi\left(1 - s_h\right) + s_h\right]^2} > 0,$$

$$B_1 = \frac{\phi s_f\left[\left(\gamma - \delta\tilde{k}^*\right)\left(1 - s_h\right) - \beta s_h\right]\left[n + s_f\left(1 - s_h\right) i\lambda^*\right]}{\left[s_f \pi\left(1 - s_h\right) + s_h\right]^2}, \quad B_2 = \frac{s_f s_h\left[n + s_f\left(1 - s_h\right) i\lambda^*\right]}{\left[s_f \pi\left(1 - s_h\right) + s_h\right]^2} > 0,$$

$$C_1 = -\frac{\phi s_f\left[\left(\gamma - \delta\tilde{k}^*\right)\left(1 - s_h\right) - \beta s_h\right]\lambda^*}{s_f \pi\left(1 - s_h\right) + s_h}, \quad C_2 = -\frac{s_f s_h \lambda^*}{s_f \pi\left(1 - s_h\right) + s_h} < 0.$$

Note that we have $\det \mathbf{J} < 0$.

The effect of an increase in the retention ratio on the capital-effective labor supply ratio can be explained as follows. Since equation (21) holds in the steady state, we obtain $\pi n - s_h i\lambda^* > 0$. Accordingly, we obtain $A_1 > 0$ when the rate of capital accumulation is debt led, $A_1 < 0$ when the rate of capital accumulation is debt burdened, and $A_2 > 0$ irrespective of whether the rate of capital accumulation is debt led or debt burdened. In addition, we obtain $J_{12} > 0$ when the rate of capital accumulation is debt led, while we obtain $J_{12} < 0$ when the rate of capital accumulation is debt burdened. Accordingly, we obtain $d\tilde{k}^*/ds_f < 0$ because we have $A_2 J_{12} - A_1 J_{22} > 0$ in the debt-led case. In contrast, we obtain $d\tilde{k}^*/ds_f > 0$ because we have $A_2 J_{12} - A_1 J_{22} < 0$ in the debt-burdened case.

Next, we obtain both $J_{12} > 0$ and $B_1 > 0$ when the rate of capital accumulation is debt led, while we obtain both $J_{12} < 0$ and $B_1 < 0$ when the rate of capital accumulation is debt burdened. Accordingly, we obtain $d\tilde{k}^*/d\pi < 0$ because we have $B_2 J_{12} - B_1 J_{22} > 0$ in the debt-led case. In contrast, we obtain $d\tilde{k}^*/d\pi > 0$ because we have $B_2 J_{12} - B_1 J_{22} < 0$ in the debt-burdened case.

In addition, we obtain both $J_{12} > 0$ and $C_1 < 0$ when the rate of capital

accumulation is debt led, while we obtain both $J_{12} < 0$ and $C_1 > 0$ when the rate of capital accumulation is debt burdened. Accordingly, we obtain $d\tilde{k}^* / di > 0$ because we have $C_2 J_{12} - C_1 J_{22} < 0$ in the debt-led case. In contrast, we obtain $d\tilde{k}^* / di < 0$ because we have $C_2 J_{12} - C_1 J_{22} > 0$ in the debt-burdened case.

Moreover, we obtain $d\tilde{k}^* / dn < 0$ because we have $J_{11} J_{22} - J_{12} J_{21} > 0$ when the rate of capital accumulation is debt led. However, the sign of $d\tilde{k}^* / dn$ is ambiguous when the rate of capital accumulation is debt burdened.

4.4 The employment rate

The steady-state value of the rate of employment is given by $e = u^* \tilde{k}^*$. Accordingly, the effect of an increase in the retention ratio on the rate of employment is give by

$$\frac{de^*}{ds_f} = \frac{du^*}{ds_f} \tilde{k}^* + \frac{d\tilde{k}^*}{ds_f} u^*. \tag{51}$$

When the rate of capital accumulation is debt led, we obtain both $du^* / ds_f < 0$ and $d\tilde{k}^* / ds_f < 0$, thereby leading to $de^* / ds_f < 0$. However, when the rate of capital accumulation is debt burdened, we obtain $d\tilde{k}^* / ds_f > 0$, and hence the effect of an increase in the retention ratio on the rate of employment is ambiguous.

From similar deductions, we can examine the effect of changes in other parameters on the rate of employment. Suppose that the rate of capital accumulation is debt led. An increase in the profit share decreases both the rate of capacity utilization and the capital-effective labor supply ratio, which decreases the rate of employment. In addition, a decrease in the interest rate lowers both the rate of capacity utilization and the capital-effective labor supply ratio, which decreases the rate of employment.

However, in the debt-burdened case, the effects of an increase in the profit share and a decrease in the interest rate on the rate of employment are ambiguous because these lower the rate of employment and raise the capital-effective labor supply ratio. In addition, the effect of an increase in the natural rate of growth on the rate of employment is ambiguous, because the sign of $d\tilde{k}^* / dn$ is ambiguous. Summarizing the above results, we obtain the following propositions.

Proposition 3. *A decrease in the retention ratio of firms worsens the financial*

structure of firms. In contrast, an increase in the profit share, a decrease in the interest rate, and an increase in the natural rate of growth improve the financial structure of firms.

Proposition 4. *A decrease in the retention ratio increases the rate of capacity utilization. In contrast, an increase in the profit share and a decrease in the interest rate decrease the rate of capacity utilization.*

Proposition 5. *Suppose the economy exhibits debt-led capital accumulation in the steady-state equilibrium. Then, a decrease in the retention ratio of firms increases both the capital-effective labor supply ratio and the rate of employment. Moreover, an increase in the profit share and a decrease in the interest rate decrease both the capital-effective labor supply ratio and the rate of employment.*

Proposition 6. *Suppose that the economy exhibits debt-burdened capital accumulation in the steady-state equilibrium. Then, a decrease in the retention ratio of firms decreases the capital-effective labor supply ratio. However, an increase in the profit share and a decrease in the interest rate increase the capital-effective labor supply ratio. Moreover, their effects on the rate of employment can be positive or negative.*

5 Changes in financial structure along the transitional dynamics

5.1 Debt-led case

Section 3.2 has analytically shown that the financial structure of firms follows a speculative finance regime in the steady-state equilibrium. However, there exists a case where the financial structure of firms goes away from the domain of speculative finance and enters into the domain of Ponzi finance along the transitional dynamics, which we will show in this section by using numerical simulations.

Let us consider the case where the rate of capital accumulation is debt led. In our numerical example, we set our parameters as follows: $n = 0.03$, $s_f = 0.8$, $s_h = 0.2$, $\pi = 0.35$, $i = 0.05$, $\alpha = 0.025$, $\beta = 0.03$, $\gamma = 0.04$, $\delta = 0.05$, and $\phi = 0.1$. In this setting, we obtain economically meaningful steady-state values such as

$g^* = 0.03$, $\lambda^* = 0.915$, and $\tilde{k}^* = 0.095$. Moreover, from $\left(\gamma - \delta\tilde{k}^*\right)(1 - s_h) - \beta s_h = 0.022$, we can easily verify that the rate of capital accumulation is debt led in the steady-state equilibrium.

Setting additionally the initial conditions to $g(0) = 0.035$, $\lambda(0) = 0.98$, and $\tilde{k}(0) = 0.3$, we obtain the transitional dynamics of the endogenous variables, which are depicted as solid lines in Figures 2, 3, and 4. As these solid lines show, all of the endogenous variables converge monotonically to their steady-state values. Furthermore, Figure 5 shows the transition of $\pi u - i\lambda$. As equation (20) indicates, the sign of $\pi u - i\lambda$ becomes negative when the financial structure of firms exhibits the Ponzi finance regime. Although $\pi u - i\lambda$ shows a sharp decline once, it does not become negative under the present setting, which implies that the financial structure of firms never falls into the Ponzi finance regime.

Now, leaving the other parameters unchanged, we raise the initial value of the capital-effective labor supply ratio from $\tilde{k}(0) = 0.3$ to $\tilde{k}(0) = 0.5$. Then, the transitional dynamics of the endogenous values change to the dashed lines in Figures 2, 3, 4, and 5. Figure 3 shows that when the initial value is given relatively far away from the steady-state value, the debt-capital ratio overshoots unity. Figure 5 shows that the sign of $\pi u - i\lambda$ becomes negative in a certain period and finally converges to the steady state in the positive area. This phenomenon is interpreted as follows. A relatively small shock in an economy keeps the financial structure of firms in the speculative finance area. By contrast, a relatively large shock in an economy temporally drives firms into the Ponzi finance regime. Firms eventually converge to the steady state in the speculative area, but it is not certain whether they can precisely predict the convergence. Unless firms can predict the convergence toward the steady state, they will not produce; if the worst happens, they would go bankrupt due to default.

[Figures 2, 3, 4, and 5 around here]

5.2 Debt-burdened case

Next, we consider the case where the rate of capital accumulation is debt burdened in the steady state. Here, we set our parameters as follows: $n = 0.015$, $s_f = 0.8$, $s_h = 0.1$, $\pi = 0.35$, $i = 0.05$, $\alpha = 0.1$, $\beta = 0.03$, $\gamma = 0.04$, and $\delta = 0.05$. We

additionally set the initial conditions to $g(0)=0.02$, $\lambda(0)=0.8$, and $\tilde{k}(0)=10$. In this example, we obtain economically meaningful steady-state values such as $g^*=0.015$, $\lambda^*=0.843$, and $\tilde{k}^*=13.998$. Moreover, we can verify that the rate of capital accumulation is debt burdened because $(\gamma-\delta\tilde{k}^*)(1-s_h)-\beta s_h=0.022$ holds.

In this setting, if the adjustment speed of the rate of capital accumulation is assumed to be $\phi=0.000735$, a limit cycle occurs under the debt-burdened case, which we have described in section 3.3. The solid lines in Figures 6, 7, and 8 show the transitional dynamics of the rate of capital accumulation, debt-capital ratio, and capital-effective labor supply ratio. Moreover, the solid line in Figure 9 indicates the transitional dynamics of $\pi u-i\lambda$, and its sign repeatedly becomes positive and negative, implying that there exists cyclical fluctuations between speculative and the Ponzi finance regime.[15]

The mechanism of the financial structure of firms to circulate between speculative and Ponzi finance is explained as follows. Suppose a shock to an economy leads to a reduction in the capital-effective labor. This would decrease the employment rate, and the firms' animal spirits would start to rise. Increases in the incentives for investment would raise the rate of capital accumulation and decrease the debt-capital ratio. Then, the financial position becomes speculative finance. However, a rise in the rate of capital accumulation has a positive impact on both the capital-effective labor supply ratio and the employment rate. When the employment rate approaches its peak, the rate of capital accumulation begins to pass through the downturn phase. A decrease in the rate of capital accumulation increases the debt-capital ratio, which in turn drives the financial position to Ponzi finance. In the situation where the debt-capital ratio is sufficiently high, the rate

[15] There is a large difference between the earlier studies that consider the dynamics of the financial structure of firms (Foley, 2003; Meirelles and Lima, 2006; Lima and Meirelles, 2007; Charles, 2008a, 2008b; and Nishi, 2012) and our model. The results of the former studies show that the firms' financial position keeps to stay in either the speculative or Ponzi finance regime once the endogenous variables converge to the steady-state equilibrium. In contrast, the key point of our model is that the financial position shows a cyclical behavior between the two regimes, because a mature economy with debt accumulation produces cyclical movements of both real and financial variables.

of capital accumulation starts to decrease due to the property of debt burden and the economy returns to its initial position, in which the capital-effective labor supply ratio is reduced.

[Figures 6, 7, 8, and 9 around here]

Furthermore, the range of fluctuations in the business cycle depends on the parameters of the model. If we decrease the retention ratio from $s_f = 0.8$ to $s_f = 0.79$ keeping all other parameters unchanged, the transitional dynamics of each variable would change from the solid lines to the dotted lines in Figures 6, 7, 8, and 9; a decline of the retention ratio would suppress the range of the cycle. One reason for this is that the absolute value of $J_{11}J_{22} - J_{12}J_{21}$, which is the instability factor of the steady state that we have shown in section 3.3, is reduced by a decrease in the retention ratio. We can use similar analysis for the other parameters; a rise in the profit share, a decrease in the interest rate, and an increase in the natural growth rate contributes to suppression of the range of fluctuation, the stability of the economy. These results are summarized in Table 2.[16]

[Table 2 around here]

6. Conclusion

This paper has presented a Kaleckian model in which both finance and labor supply constraints influence economic growth. Using this model, we have investigated analytically the impacts of changes in the retention ratio, profit share, rate of interest, and natural rate of growth on capital/debt accumulation and on firms' financial structure in a mature economy. Our results are summarized as

[16] Note here that our numerical simulation results are obtained on the basis of plausible parameter constellations. As we described above, the range of fluctuation depends on the absolute value of $J_{11}J_{22} - J_{12}J_{21}$. However, we cannot find the analytical results of the impact of a change in a parameter on the absolute value of $J_{11}J_{22} - J_{12}J_{21}$ because of complicated calculations.

follows.

Stability analysis. If the rate of capital accumulation exhibits a debt-led regime, the steady-state equilibrium is locally stable. On the other hand, if the rate of capital accumulation exhibits a debt-burdened regime, there exist three cases: (1) the steady-state equilibrium is locally stable, (2) the steady-state equilibrium is locally unstable, and (3) a limit cycle occurs when the adjustment speed of the rate of accumulation approaches to some value.

Comparative statics analysis. A decrease in the retention ratio raises the rate of capacity utilization, whereas it makes the financial structure fragile. Moreover, it increases the capital-effective labor supply ratio and employment rate in the debt-led case. However, a decline in the retention ratio reduces the capital-effective labor supply ratio in the debt-burdened case; consequently, the effect of a decrease in the retention ratio on employment is ambiguous. Moreover, an increase in the profit share and a decrease in the interest rate reduce the rate of capacity utilization and improve the financial position of firms. Then, these lead to a decline in the capital-effective labor supply ratio and employment rate in the debt-led case. In contrast, an increase in the profit share and a decrease in the interest rate raise the capital-effective labor supply ratio, and accordingly these have an ambiguous effect on the employment rate in the debt-burdened case. Finally, a rise in the natural rate of growth caused by increases in the labor productivity growth and labor supply growth has a positive impact on the financial structure of firms, although its impact on the rate of capacity utilization, capital-effective labor supply ratio, and rate of employment is not clear, at least from an analytical point of view.

Transitional dynamics of financial structure. In the steady-state equilibrium, the financial structure of firms becomes speculative finance irrespective of whether the economy exhibits debt-led or debt-burdened growth. However, there is a difference in the transitional dynamics of the financial position between the debt-led case and debt-burdened case. Our numerical simulations have revealed that in the debt-led case, a relatively large shock brings on an overshoot to the debt-capital ratio, and accordingly the financial structure temporally goes into a Ponzi finance regime; on the other hand, if there occurs a limit cycle in the debt-burdened case, the financial position circulates between speculative finance

and Ponzi finance regimes. Furthermore, we have clarified that decreases in the retention ratio and interest rate and increases in the profit share and natural rate of growth suppress the range of cyclical fluctuations.

Finally, we sum up the total effects of pro-shareholder income redistribution on the macroeconomic performance in the financialization era. As we have shown, a decrease in the retention ratio has both advantages and disadvantages; it worsens the financial structure of firms, but stabilizes the economy. In contrast, a rise in the profit share contributes to the financial position as well as the stability of the economy. Thus, we are inclined to conclude that an increase in the profit share is more effective for the economic performance than a decline in the retention ratio. However, raising the profit share is not so easy, because it produces inconsistent redistribution for households in that a rise in the profit share may lead to a decrease in the wage income and an increase in dividend income. If the number of capitalist-type households who hope for an increase in the dividends are larger than worker-type households who prefer a rise in the wage income, increases in the profit share is likely to be realized. Otherwise, it is not likely. Therefore, the total assessment of income distribution under financialization largely depends on the orientation of firms and choice of households.

Appendix: A sufficient condition for fulfilling Assumption 3

In what follows, we show that when the trend term of the target rate of capital accumulation α is sufficiently large, Assumption 3 (i.e., $a_2 > 0$) is fulfilled. One of the coefficients of the characteristic equation, a_2, is given by

$$\begin{aligned} a_2 &= J_{11}J_{22} - J_{12}J_{21} - J_{13}J_{31} \\ &= \frac{\phi\left[s_f\pi(1-s_h-\beta)+s_h-\gamma+\delta\tilde{k}^*\right]\left\{\left[s_f\pi(1-s_h)+s_h\right]g^* - s_f s_h i\right\}}{\left[s_f\pi(1-s_h)+s_h\right]^2} \\ &\quad - \frac{\phi s_f i\left[\left(\gamma-\delta\tilde{k}^*\right)(1-s_h)-\beta s_h\right]\left\{s_h(1-s_f\pi)-\left[s_f\pi(1-s_h)+s_h\right]\lambda^*\right\}}{\left[s_f\pi(1-s_h)+s_h\right]^2}. \\ &\quad + \frac{\phi\delta\tilde{k}^*\left[g^* + s_f(1-s_h)i\lambda^*\right]}{s_f\pi(1-s_h)+s_h} \end{aligned} \tag{A1}$$

From equation (16), we have $d\tilde{k}^*/d\alpha > 0$. Thus, the larger α is, the larger the first term of the right-hand side in equation (A1). The second and third terms can be rewritten as follows:

$$-J_{12}J_{21}-J_{13}J_{31}=\phi\frac{s_f i\left[\gamma\left(1-s_h\right)-\beta s_h\right]\left\{\left[s_f\pi\left(1-s_h\right)+s_h\right]\lambda^*-s_h\left(1-s_f\pi\right)\right\}}{\left[s_f\pi\left(1-s_h\right)+s_h\right]^2}$$
$$+\phi\frac{\delta\tilde{k}^*\left\{\left[s_f\pi\left(1-s_h\right)+s_h\right]g^*+s_f s_h i\left(1-s_h\right)\left(1-s_f\pi\right)\right\}}{\left[s_f\pi\left(1-s_h\right)+s_h\right]^2}. \quad \text{(A2)}$$

The sign of the first line of the right-hand side in equation (A2) is ambiguous, although the sign of the second line is strictly positive. From $d\tilde{k}^*/d\alpha>0$, if α is sufficiently large, the second line becomes large and dominates the sign of the right-hand side; in other words, $-J_{12}J_{21}-J_{13}J_{31}$ is likely to become positive. Thus, a sufficiently large α satisfies Assumption 3.

References

Abe, T. "Technical Progress and Maturity in a Kaleckian Model of Growth with an Endogenous Employment Rate." *Nagoya Gakuin University Institute Discussion Paper* no. 89 (2012).

Bhaduri, A., and S. Marglin. "Unemployment and the Real Wage: The Economic Basis for Contesting Political Ideologies." *Cambridge Journal of Economics* 14, no. 4 (1990): 375–393.

Blecker, R.A. "Distribution, Demand and Growth in Neo-Kaleckian Macro-Models," in Setterfield, A. ed. *The Economics of Demand-led Growth: Challenging the Supply-side Vision of the Long Run*. Cheltenham: Edward Elgar, 2002.

Charles, S. "A Post-Keynesian Model of Accumulation with a Minskyan Financial Structure." *Review of Political Economy* 20, no. 3 (2008a): 319–331.

Charles, S. "Teaching Minsky's Financial Instability Hypothesis: A Manageable Suggestion." *Journal of Post-Keynesian Economics*, 31, no. 1 (2008b): 125–138.

Dallery, T. "Post-Keynesian Theories of the Firms under Financialization." *Review of Radical Political Economics* 41, no. 4 (2009): 492–515.

Epstein, G.A., and D. Power. "Rentier Incomes and Financial Crises: An Empirical Examination of Trends and Cycles in Some OECD Countries." *Institute for International Political Economy Working Paper* no. 57 (2003).

Foley, D. "Financial Fragility in Developing Countries," in Dutt, A.K., and J. Ross eds. *Development Economics and Structuralist Macroeconomics: Essays in*

Honor of Lance Taylor, Aldershot: Edward Elgar, 2003.

Fujita, S., and H. Sasaki. "Financialization and its Long-Run Macroeconomic Effects in a Kalecki-Minsky Model." *Kyoto University, Graduate School of Economics Research Project Center Discussion Paper*, no. E-11-001 (2011).

Hein, E. "Interest, Debt and Capital Accumulation: A Kaleckian Approach." *International Review of Applied Economics* 20, no. 3 (2006): 337–352.

Hein, E. "Interest Rate, Debt, Distribution, and Capital Accumulation in a Post-Kaleckian Model." *Metroeconomica* 56, no. 2 (2007): 310–339.

Hein, E. "Shareholder Value Orientation, Distribution and Growth—Short- and Medium-Run Effects in a Kaleckian Model." *Metroeconomica* 61, no. 2 (2010): 302–332.

Hein, E. "'Financialization,' Distribution, Capital Accumulation, and Productivity Growth in a Post-Keynesian Model." *Journal of Post-Keynesian Economics* 34, no. 3 (2012): 475–496.

Hein, E., and T. van Treeck. "Financialization and Rising Shareholder Power in Kaleckian/Post-Kaleckian Models of Distribution and Growth." *Review of Political Economy* 22, no. 2 (2010a): 205–233.

Hein, E., and T. van Treeck. "'Financialization' in Post-Keynesian Modes of Distribution and Growth: A Systematic View," in Setterfield, M., ed. *Handbook of Alternative Theories of Economic Growth.* Cheltenham: Edward Elgar, 2010b.

Kalecki, M. *Theory of Economic dynamics.* London: George Allen, 1954.

Lavoie, M. "Interest Rate in Post-Keynesian Models of Growth and Distribution." *Metroeconomica* 46, no. 2 (1995): 146–177.

Lima, G.T., and A.J.A. Meirelles. "Macrodynamics of Debt Regimes, Financial Instability and Growth." *Cambridge Journal of Economics* 31, no. 4 (2007): 563–580.

Meirelles, A.J.A., and G.T. Lima. "Debt, Financial Fragility, and Economic Growth: A Post Keynesian Macromodel." *Journal of Post Keynesian Economics* 29, no. 1 (2006): 93–115.

Minsky, H. *John Maynard Keynes.* New York: Columbia University Press, 1975.

Minsky, H. *Can "It" Happen Again?: Essays on Instability and Finance.* New York: M. E. Sharpe, 1982.

Moore, B.J. *Horizontalists and Verticalists: The Macroeconomics of Credit Money.*

Cambridge: Cambridge University Press, 1988.

Nishi, H. “A Dynamic Analysis of Debt-led and Debt-burdened Growth Regimes with Minskian Financial Structure,” forthcoming in *Metroeconomica.* DOI: 10.1111/j.1467-999X.2012-04158.x (2012).

Onaran, Ö., Stockhammer, S., and L. Grafl. “Financialisation, Income Distribution and Aggregate Demand in the USA.” *Cambridge Journal of Economics* 35, no. 4 (2011): 637–661.

Orhangazi, Ö “Financialization and Capital Accumulation in the Non-financial Corporate Sector: A Theoretical and Empirical Investigation on the US Economy: 1973–2003.” *Cambridge Journal of Economics* 32, no. 6 (2008): 863–886.

Rochon, L.P. *Credit, Money, and Production: An Alternative Post-Keynesian Approach.* Cheltenham: Edward Elgar, 1999.

Ryoo, S., and P. Skott. “Financialization in Kaleckian Economies with and without Labor Constraints.” *Intervention. European Journal of Economics and Economic Policies* 5, no. 2 (2008): 357–386.

Sasaki, H., and S. Fujita. “The Importance of the Retention Ratio in a Kaleckian Model with Debt Accumulation.” *Metroeconomica* 63, no. 3 (2012): 417–428.

Skott, P., and S. Ryoo. “Macroeconomic Implication of Financialization.” *Cambridge Journal of Economics* 32, no. 6 (2008): 827–862.

Skott, P., and B. Zipperer. “An Empirical Evaluation of Three Post-Keynesian Models,” forthcoming in *Intervention. European Journal of Economics and Economic Policies*.

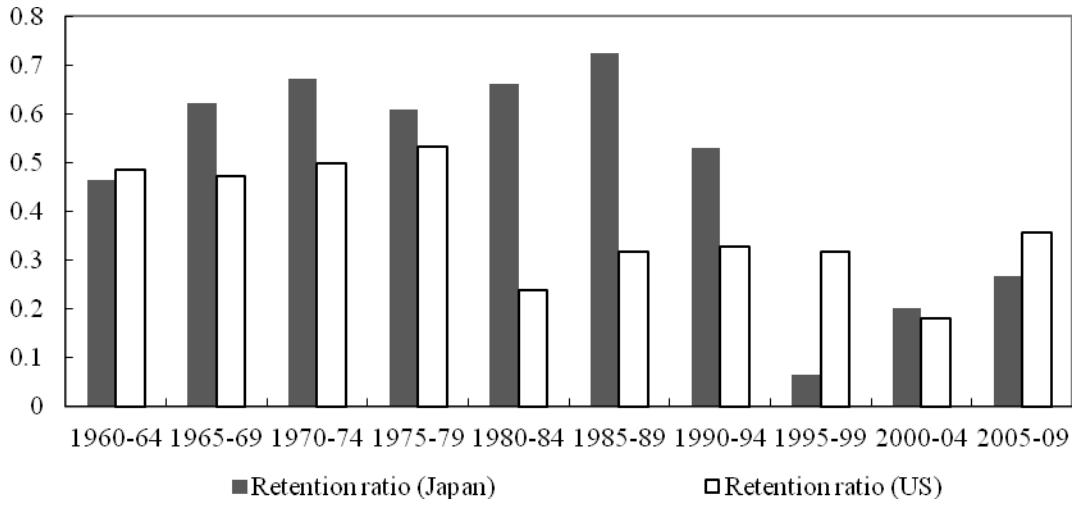
Stockhammer, S. “Financialization and Accumulation.” *Cambridge Journal of Economics* 28, no. 5 (2004): 719–741.

Taylor, L. *Reconstructing Macroeconomics: Structuralist Proposals and Critiques of the Main Stream.* Massachusetts: Harvard University Press, 2004.

van Treeck, T. “Reconsidering the Investment–Profit Nexus in Financed-led Economies: An ARDL-based Approach.” *Metroeconomica* 59, no. 3 (2008): 371–404.

Figures and Tables

Figure 1: The retention ratio in Japan and the U.S. (1960–2009)



Source: Financial Statements Statistics of Corporations by Industry, Policy Research Institute, Ministry of Finance, Japan. National Income and Product Accounts, Bureau of Economic Analysis, U.S. Department of Commerce.

Figure 2: Transition of rate of capital accumulation in the debt-led case

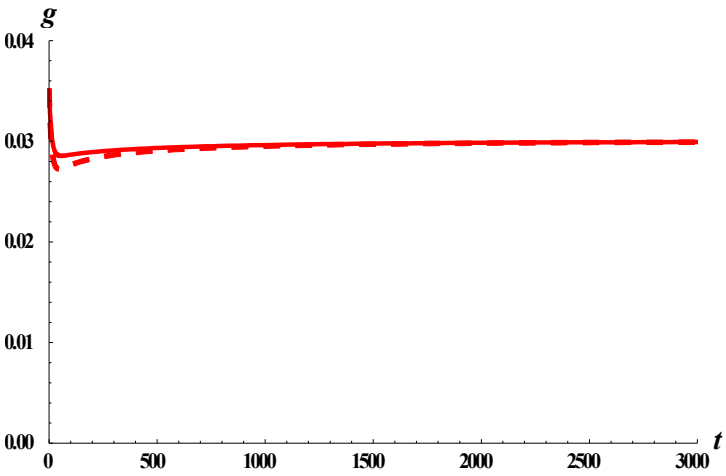


Figure 3: Transition of debt-capital ratio in the debt-led case

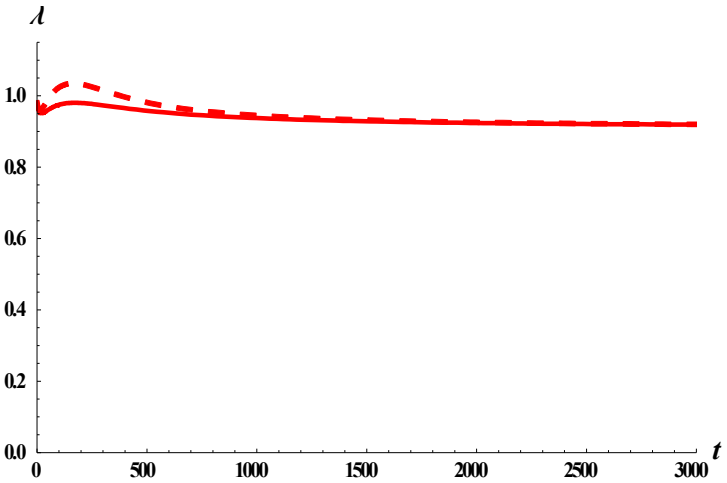


Figure 4: Transition of capital-effective labor supply ratio in the debt-led case

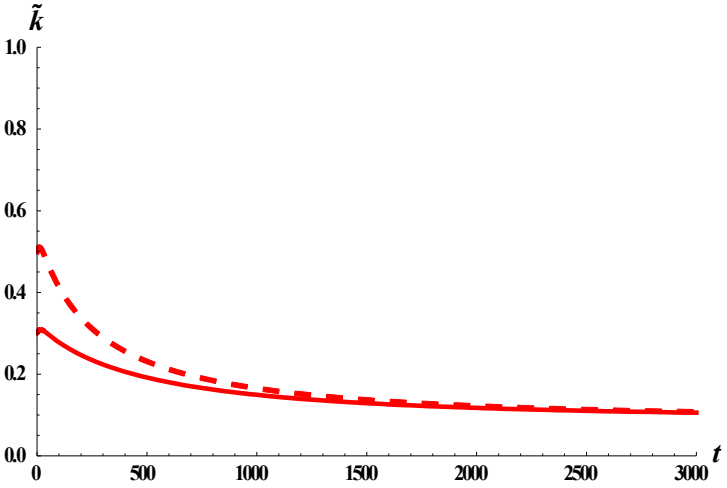


Figure 5: Transition of $\pi u - i\lambda$ in the debt-led case

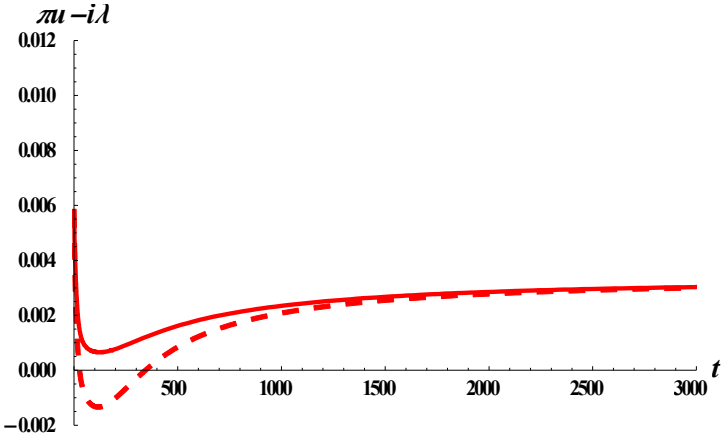


Figure 6: Transition of rate of capital accumulation in the debt-burdened case

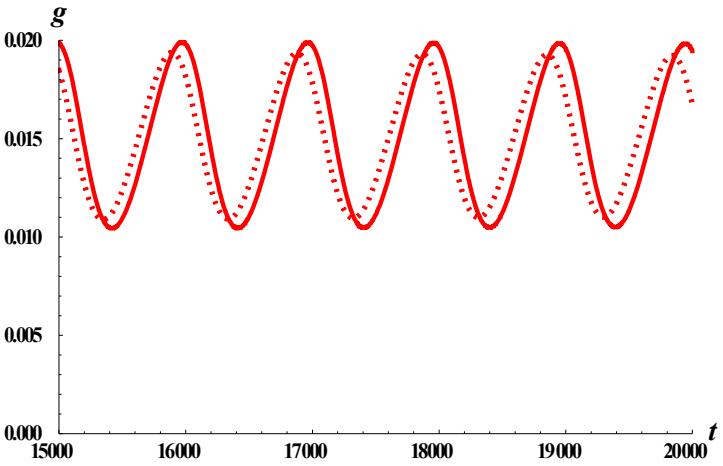


Figure 7: Transition of debt-capital ratio in the debt-burdened case

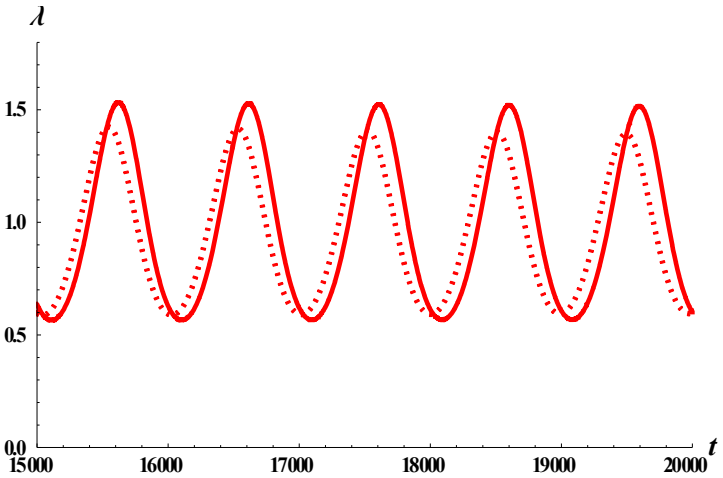


Figure 8: Transition of capital-effective labor supply ratio in the debt-burdened case

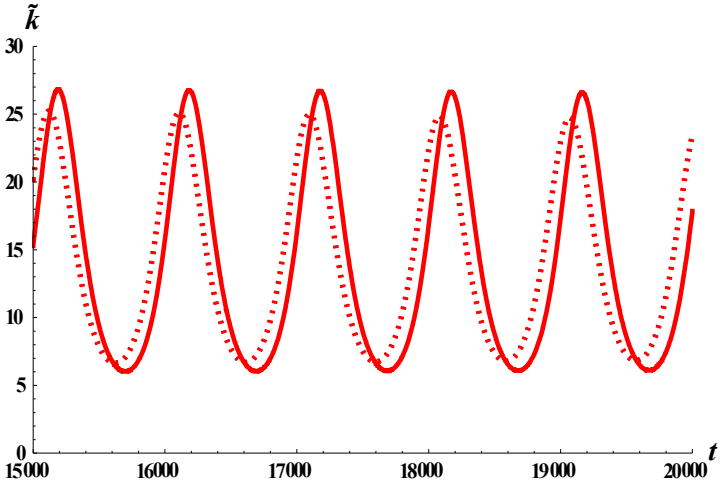


Figure 9: Transition of $\pi u - i\lambda$ in the debt-burdened regime

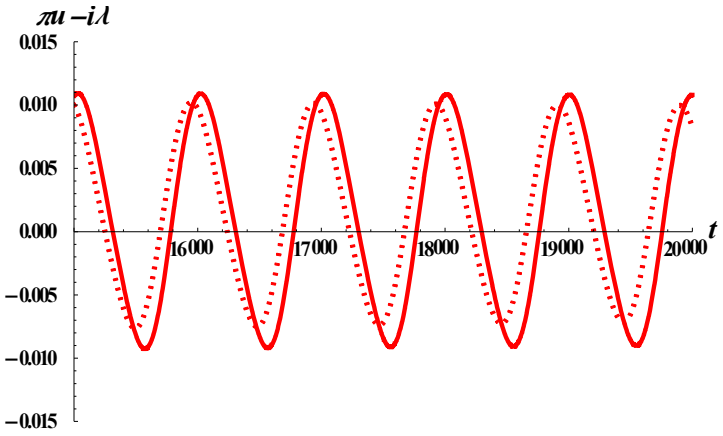


Table 1: Taxonomy of the financial structures of firms

Finance regime	Definition of each finance regime	Steady-state equilibrium
Hedge	$\pi u \geq g + i\lambda + (1 - s_f)(\pi u - i\lambda)$	No
Speculative	$\pi u < g + i\lambda + (1 - s_f)(\pi u - i\lambda)$ and $\pi u > i\lambda + (1 - s_f)(\pi u - i\lambda)$	Yes
Ponzi	$\pi u \leq i\lambda + (1 - s_f)(\pi u - i\lambda)$	No

Table 2: The effect of parametric changes on the range of fluctuation

	s_f	π	i	n
Range of fluctuation	+	-	+	-