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Abstract

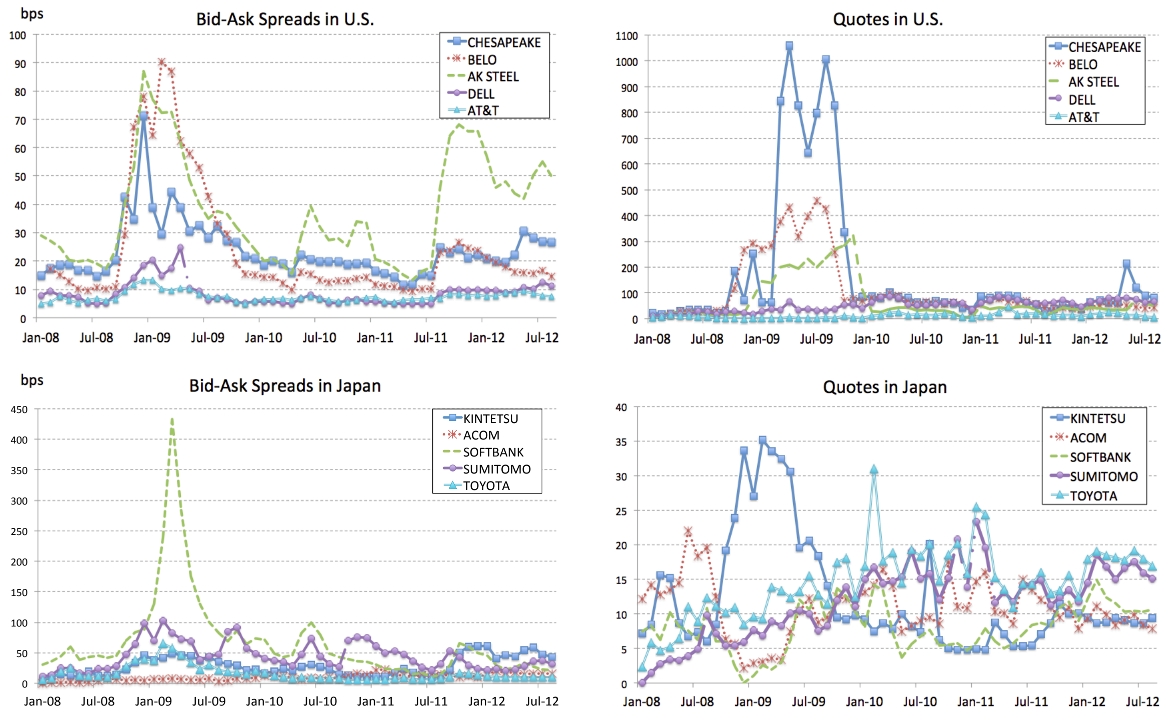
We study the credit default swap (CDS) markets in the U.S. and Japan, focusing on bid-ask spreads which are closely related to the liquidity of the markets. Since bid-ask spreads dramatically surged during the financial crisis (2008-2009) and the market became very illiquid, it is crucially important to investigate how bid-ask spreads fluctuate. In this paper, not only do we make dynamic analysis of the bid-ask spreads in both countries but propose a model to predict bid-ask spreads via the self-exciting intensity process (the Hawkes process).

Key words: CDS contract; liquidity; bid-ask spread; the Hawkes process; self-exciting processes, financial crisis, credit risk.
JEL Classification: G10, G17, G21

1 Introduction

The credit default swap (CDS) contract is a major single-name credit derivative and serves as the building block for many other credit derivative instruments. We refer to O'Kane [13] for the definition of the CDS contract: A CDS is a bilateral over-the-counter contract whose purpose is to protect one party, the *protection buyer*, from the loss from par on a specified face value of bonds, or loans following the default of their issuer. The CDS markets expanded rapidly during the first half of the last decade. We have witnessed, however, a huge pike of bid-ask spreads in the U.S. CDS market during the financial crisis period. It is said that the *liquidity* evaporated and the market participants had difficulty in completing their transactions. See the bid-ask spreads ("BAS" hereafter) and the number of quotes in the U.S. and Japan CDS markets in Figure 1.

Figure 1: Daily averages of BAS and Quote numbers



We shall focus on BAS in this paper for the following reasons: First, Figure 1 shows that, during the crisis period, BAS increased drastically in both U.S. and Japan. It is obvious that the variability of BAS is closely related to the liquidity of the CDS markets. In this sense, we believe that it is crucial to understand in what way the spreads fluctuate, sometimes vastly, and to know mechanism behind it. Second, in view of Longstaff et al. [10], the CDS premia are largely explained by credit component, so the information about liquidity of the market is mainly contained in BAS. Of course, however, market makers often bear underling credit risk during their holding period until they sell the contract. Thus, if the default probability of the underling loan/bond goes up, they may end up with holding an open position, so that BAS also contains, to some extent, credit risk information. But again, this increased credit risk level is promptly reflected in the rise of CDS premia, and hence the credit part of BAS seems to vanish soon or later, as long as there is enough liquidity is provided in the market. Also, it is worth mentioning that Roll [15] claims in the study of the stock market that BAS are efficiently determined by market makers through their functioning as liquidity providers to the market. We therefore investigate

Table 1: Comparison of the CDS indices between U.S. and Japan

(million $, 2009)	gross notional	net notional	# contracts
CDX.NA.IG SERIES 9	1,324,894	75,213	22,334
iTraxx Japan SERIES 11	13,032	1,448	1,050

(million $, 2010)	gross notional
Japan	1,116,900
World	30,261,000

the CDS markets in the U.S. and Japan and fit some models to explain fluctuations of the BAS.

Studying both the U.S. and Japan markets is beneficial because the degree of market efficiency differ. Table 1 is a comparison of the contracts of the CDS indices, indicating a big difference in size.

In particular, an observation of time series data of CDS premia reveals the following: Larger and more frequent price swings have been recorded in the U.S. market. Lower-grade names are more frequently traded than investment-grade names in the U.S., while trade volumes in higher-grade names are far greater (than lower-grade) in Japan. Since the Japanese market is in its early period, a full utilization of the market to trade credit risks may have not been accomplished. It seems instructive to analyze the "developed" and "developing" markets simultaneously since one can understand the process of market evolution as well as the similarity and difference of the two markets.

Now let us take a look at Figure 1 again for the time series data of BAS and the numbers of quotes. First, the number of quotes are far greater in the U.S. market compared to Japan. In both countries, as we know, there are high peaks in the quote numbers and BAS during the financial crisis (2008 $\sim$ 2009)[1]. The relationship between these two variables needs more investigation: we shall also see the non-crisis periods in developing a model that predicts BAS.

We shall briefly mention main results of this paper:

(1) In Section 1.1, by simple regressions, changes in BAS of U.S. companies' CDS contracts can be explained both by their respective bid prices and the economic factors that surround the companies. This is not the case in Japan, where the CDS market does not fully incorporate general and/or company-specific economic factors. The difference in the market efficiency may be attributable to this contrasting phenomenon.

[1] A special mention is necessary for a huge upsurge in the bid-ask spread of Softbank contract: it is due to its announced merger of the struggling mobile provider, Willcom.

(2) We observe time-series data of bid price and BAS in Section 2.1. By the Grander causality test, we find that the two variables have causality in both directions: Bid price is a cause of BAS changes and vice versa. Moreover, there exists a "reversal" phenomenon in BAS changes. If one observes an increase in BAS at some point, then in the next period BAS has a tendency to decline.

(3) Over a long period (Jan 2002 through 2010) there is a large variability in correlation coefficients between bid prices and BAS (in Section 2.2).

(4) We construct a model to explain BAS movements based on quote numbers. The main purpose of this modeling is to predict near-future BAS, which we believe is of practical use. We observe a short-run surge and fall in the correlation coefficient between the quote numbers and BAS. More specifically, we attempt to model the number of quotes by a self-exciting intensity process (or the Hawkes process). The term "self-exciting" implies that the intensity of quote arrivals to the market makers increases as quote numbers themselves increase. We then use the time-changed Brownian motion to model BAS fluctuations. The new clock here is arrival times of quotes. See (3.1)$\sim$(3.3). The prediction results shown in Section 3 are very promising. From what we observe in the two performance tests, the Hawkes process-based prediction does pretty well, whether it is used for the U.S. or Japanese market, and whether it is used for higher or lower credit rating companies.

There are numerous empirical studies on how CDS prices are determined. Blanco et al. [2] find the impact of firm-specific stock returns is stronger on CDS price changes than on corporate bond spread changes. Norden and Weber [11] report, among other things, that the lower company's credit rating, the more sensitive CDS prices to its stock price, and that the CDS market contributes more to price discovery than the bond market and this effect is stronger for U.S. than for European companies. Acharya and Johnson [1] provide empirical evidence that there is an information flow from the credit default swap markets to equity markets and the flow is concentrated on days with negative credit news. More specific to liquidity, Longstaff et al. [10] report that the majority of the corporate spread is due to default risk (rather than liquidity risk), which result holds for all rating categories. Moreover, they find the nondefault component is time varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond market liquidity. In this vein, more recently, Bühler and Trapp [3] propose a reduced-form model to decompose bond spreads and CDS premia into three components (pure credit risk, liquidity risk and a component measuring the reltion between credit and liquidity risk components). Ericsson and Renault [6] report that in finite maturity debt markets, there exist decreasing and convex term structures of liquidity. The Hawkes process we shall employ in this paper is proposed by Hawkes [8]. In the finance literature, there are a number of papers that model certain economic variables by using the Hawkes processes. For example, Hewlett [9] uses for order arrivals in the stock market and Errais et al. [7] use for default events in a credit-related portfolio of large size.

We mention the data we use in this paper. For the regression analysis in section 1.1, vector autoregression in section 2.1 and DCC-GARCH in section 2.2.1, we take all the information including bid prices and bid-ask spreads from DATASTREAM by Thomson Reuters. The period is January 2003 to September 2010. Since the quote numbers are not available in DATASTREAM we subscribe, we obtain quote numbers and bid-ask spreads from Markit Group Limited. The period is January 2008 to August 2012. (Note that the data of quote numbers are available only after January 2008.) Quote numbers are used (along with bid-ask spreads) for DCC-GARCH in section 2.2.2 and for the analysis based on the Hawkes process in sections 3.1 $\sim$ 3.2. Note that all the tables from the statistical analysis are placed in Appendix.

1.1 Preliminary Regression Analysis

Bid vs Economic Variables: We shall take a look at "Bid" price of the CDS via a simple regression. This is because the BAS may be influenced by bid prices.

$$\begin{aligned}\Delta(\text{Bid})_t = a_0 + b_1\Delta(\text{Interest})_t &+ b_2\Delta(\text{VI})_t + b_3\Delta(\text{Volume}) \\ &+ c_1\Delta(\text{StockPrice})_t + c_2\Delta(\text{StockSigma})_t + c_3\Delta(\text{StockVol})_t + d_1(\text{Crisis})_t + \epsilon_t. \qquad (1.1)\end{aligned}$$

The objective variable is $\Delta(\text{Bid})_t$ is the difference in bid prices at time t. The first three valuables are, in essence, related to macro-economic situations: $\Delta(\text{Interest})_t$ is the change in yield rates of 10-year government bonds, $(\text{VI})_t$ is the change in VIX (U.S.) or Nikkei Volatility Index, $\Delta(\text{Volume})_t$ is the change in trading volumes in the Dow Jones Index or TOPIX. The next three valuables are specific to the underlying loan/bond issuer: $\Delta(\text{StockPrice})_t$ is the change in the issuer's stock price, Δ(StockSigma) is the change in the stock price volatility from the last 4 prior weeks, and Δ(StockVolume) is the change in trading volume of the issuer's stock. Finally, we attach a dummy variable (Crisis) taking a value of zero if t is prior to September 15, 2009 and of unity if t is after that date. The result is summarized in Table 3 in the Appendix.

The third and fourth columns of the upper and lower panel are the adjusted R^2 of regression (1.1) for the U.S. and Japanese markets, respectively. In the U.S. market, the average (over 20 names) R^2's are 0.1308 in the one-year CDS and 0.1988 in the five-year CDS. Several variables such as $\Delta(\text{VI})_t$, $\Delta(\text{StockPrice})_t$ $\Delta(\text{StockSigma})_t$ have significant non-zero coefficients. (See Table 4.) In contrast, in Japan, the average (over 20 names) R^2 are 0.01324 in one year CDS and 0.03966 in five year contract, far below the U.S. counterparts. The three valuables that seem to have significant impact in the U.S. market are not the cases in Japan. To get more information, we separate Japanese corporate names into two categories: high credit rating (A- or greater by the Japanese rating agency R & I) and low credit rating (BBB+ or lower). The average R^2's are improved in the high rating category: one year CDS, 0.02471 and five year 0.05504, while the low rating category become even worse: one year CDS, 0.001767 and five year, 0.02429.

It is clear that part of variations of the bid prices in the U.S. market can be explained by the micro- and macroeconomic factors. But in Japan, bid prices are not so well explained by these factors, especially in the low rating category. In both markets, the R^2 are better in five year contracts than one year.

BAS vs Bid: The next question is to what extent BAS can be explained by bid prices:

$$\Delta(\text{BAS})_t = a_0 + a_1\Delta(\text{Bid})_t + d_1(\text{Crisis}) + \epsilon_t. \tag{1.2}$$

The results are shown in the fifth and sixth columns of Table 3. The averages (over 20 names) in Japan are 0.09647 (1 year) and 0.1093 (5 year) with significant non-zero estimates of a_1 in almost all names. In the U.S. market, the numbers are 0.08024 (one year) and 0.16369 (five year) with significant non-zero estimates, too.

BAS vs (Bid + Economic Variables): A natural question is then if we add six variables in (1.1) to the estimation (1.2), can we get any improvement?

$$\begin{aligned}\Delta(\text{BAS})_t = a_0 + a_1\Delta(\text{Bid})_t &+ b_1\Delta(\text{Interest})_t + b_2\Delta(\text{VI})_t + b_3\Delta(\text{Volume})\\ &+ c_1\Delta(\text{StockPrice})_t + c_2\Delta(\text{StockSigma})_t + c_3\Delta(\text{StockVol})_t + d_1(\text{Crisis})_t + \epsilon_t.\end{aligned} \tag{1.3}$$

To answer the question, we conduct the partial F-test: See the last two columns of Table 3. The marginal contribution of the six variables to R^2 is tested. In the U.S. market, the six additional variables have statistically significant contribution in 9 names out of 20 names at 5 % significant level. Especially, 7 names (in 1 year contracts) and 6 names (in 5 year) out of 10 names in low rating category show significant improvement. Since, in the U.S., the CDS market is more liquid in the lower grade category and is used for hedging credit risk of these companies, the level of BAS incorporates many factors. Norden and Weber [11] report that the higher the credit risk, the more sensitive to stock return fluctuations the CDS price becomes. Our result here is consistent with this: Since a lower-graded company is more exposed to "flight to liquidity", market makers may promptly respond to changes in general and/or company-specific factors by widening and shrinking BAS. In Japan, in contrast, only in 4 names (mostly higher grade) out of 20 names do we observe marginal improvements. The latter result is consistent with the regression result of (1.1).

As a consequence of this test, we may say that in the U.S. market, BAS is explained both by bid prices and by micro-and macroeconomic factors, while in the Japanese market, BAS is explained only by bid prices. Since the CDS market in Japan is in its incipient stage, BAS are charged for some reasons that do not necessarily reflect the general economic or company specific factors. In particular, in the lower grade category, this phenomenon is more apparent.

2 Time Series Analysis

2.1 Autoregressive Model

To make a further comparison between the U.S. and Japanese markets and to better understand the interaction between bid prices and BAS, we consider the following vector autoregression (VAR): We tested 20 names in each market, using the data from Jan-2003 to Sep-2010.

$$
\begin{cases} \Delta(\mathrm{Bid})_t = a_1 + b_{11}\Delta(\mathrm{Bid})_{t-1} + b_{12}\Delta(\mathrm{BAS})_{2,t-1} + \epsilon_{1,t}, \\ \Delta(\mathrm{BAS})_t = a_2 + b_{21}\Delta(\mathrm{Bid})_{t-1} + b_{22}\Delta(\mathrm{BAS})_{2,t-1} + \epsilon_{2,t} \end{cases} \tag{2.1}
$$

where $\begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}$ has the variance-covariance matrix $\begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$. The results are shown in Table 5. Let us first see the signs of each coefficient. The most striking feature is that the sign of b_{22} (ΔBAS$\rightarrow$ ΔBAS) is negative in both markets and in both 1 year and 5 year contracts, while the sign of b_{21} (ΔBid$\rightarrow$ ΔBAS) is positive in most of the cases. Hence if one observes an increase in CDS price at some period, the next period is likely to witness an increase in BAS. Regarding this point, it is interesting to observe 5 out of 20 U.S. companies have negative signs on Δ(Bid) variables in regression (1.3), while 9 out of 20 U.S. companies have positive signs on the variable. See Table 4.

On the other hand, if one observes an increase in BAS at some point, then one may see a decline in BAS in the next period. This "reversal" property checks BAS not to keep rising, so BAS remains in reasonable stable levels.

Another point to make is that there exists strong Granger causality from ΔBAS to ΔBid in 11 names (6 names) out of 20 in the US 1 year (resp. 5 year) CDS market. There are also 9 names in 1 year contracts and 8 names in 5 year contracts in Japanese market. This causality is a bit counterintuitive. A possible explanation is that the market participants may take BAS into consideration when they determine absolute levels of credit risk (i.e., bid price). To get more insights into this phenomenon, we conducted more Grander causality tests including quote numbers (Quotes, hereafter): See Table 6 where we test $\Delta(\text{Bid}) \rightleftarrows \Delta(\text{Quotes})$ and $\Delta(\text{BAS}) \rightleftarrows \Delta(\text{Quotes})$ and report the F-statistics. The causality observed here is less significant than the causality from Δ(BAS) to Δ(Bid). Hence if we look at a long time period, the relationship between the bid prices and BAS is stronger than that between Quotes and BAS.

2.2 DCC-GARCH Model

2.2.1 BAS-Bid

Next, we study dynamic correlation models with GARCH$(1,1)$, following the method proposed by Engel [5]. A brief explanation of this model is as follows: Defining an appropriate filtration $(\mathcal{F}_t)_{t\geq 0}$, the n-dimensional vector at time t, y_t is modeled by $y_t = \mu_t + a_t$ where $\mu_t = \mathbb{E}(y_t|\mathcal{F}_{t-1})$ and $a_t = H_t^{1/2} v_t$ with $v_t \sim$ i.i.d$N(0, I_n)$. We then have $H_t = \text{Cov}(a_t|\mathcal{F}_{t-1})$. Now H_t is in turn modeled by

$$H_t = D_t R_t D_t$$

where $R_t = (\rho_{ij,t})_{n\times n}$ is a positive definite matrix, $D_t = \text{diag}\{\sqrt{h_{11,t}}, \dots, \sqrt{h_{nn,t}}\}$. Moreover, R_t is

$$R_t = \text{diag}(Q_t)^{-\frac{1}{2}} Q_t \text{diag}(Q_t)^{-\frac{1}{2}}$$

where Q_t satisfies

$$Q_t = (1-\theta_1-\theta_2)\bar{Q} + \theta_1 \epsilon_{t-1}\epsilon'_{t-1} + \theta_2 Q_{t-1}.$$

Here ϵ_t is the standardized innovation vector with $\epsilon_t = D_t^{-1} a_t$ and $\bar{Q}$ is the unconditional correlation matrix of ϵ_t. For Q_t to be positive definite, a sufficient condition is that $\theta_1, \theta_2 > 0, \theta_1 + \theta_2 \leq 1$.

We fit the above DCC-GARCH(1, 1) to 10 names (5 each in the U.S. and Japan)

(1) Bid - BAS: from Jan-2003 to Sep-2010 (Figure 2) and

(2) BAS - Quotes: from Jan-2008 to Aug-2012 (Figure 3).[2]

First, from Figure 2, we observe large variability in correlation coefficients (between Bid price and BAS) in general. This is consistent with the causal relationship between the two variables confirmed in Table 5. In the U.S. market, however, the fluctuations seem greater as the credit rating goes worse. Compare AT&T with credit rating A- and Chesapeake Energy (BB-). In contrast, in the Japanese

[2]The quote numbers are available only after January 2008.

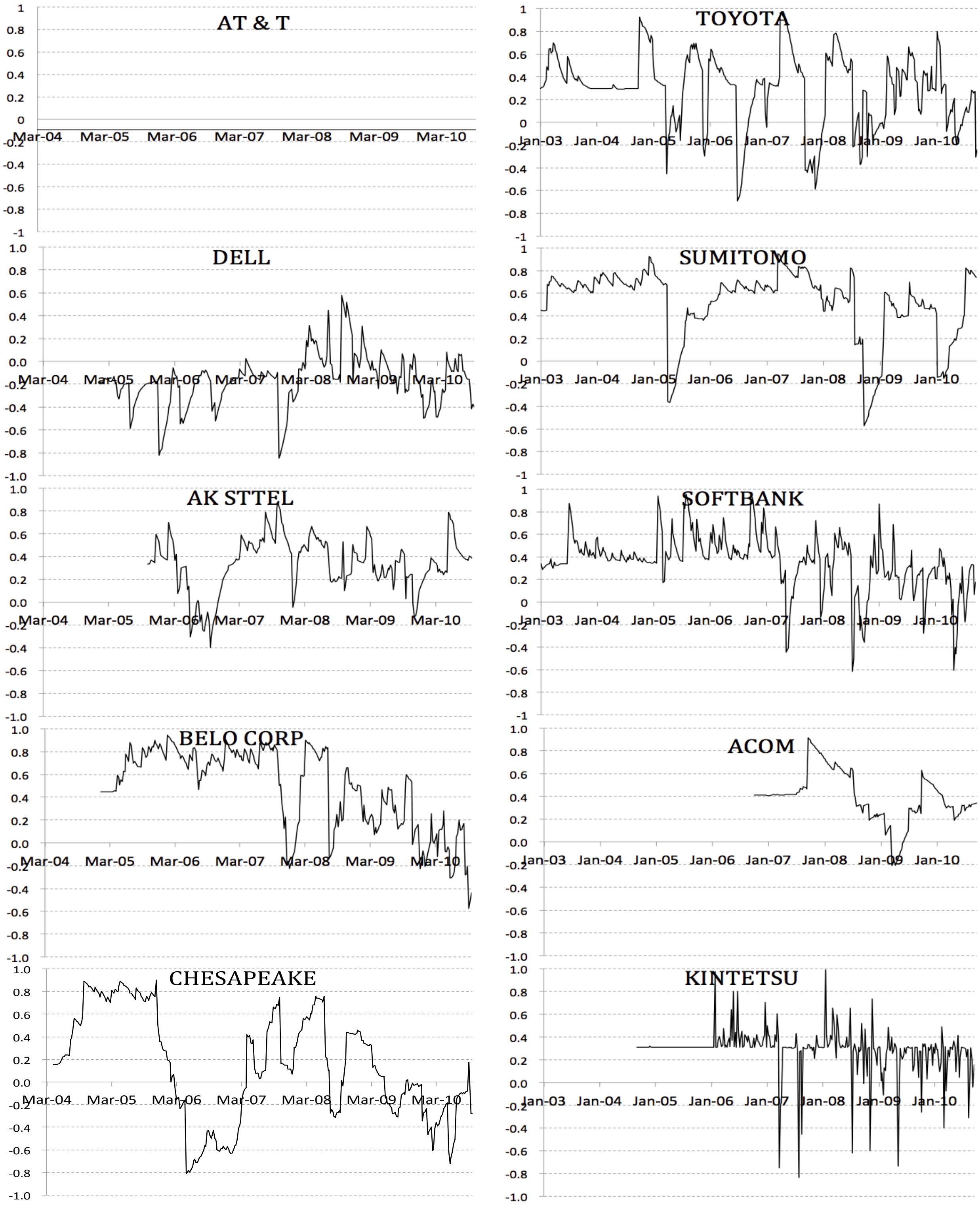


Figure 2: Estimated dynamic correlations between BAS-Bid Price

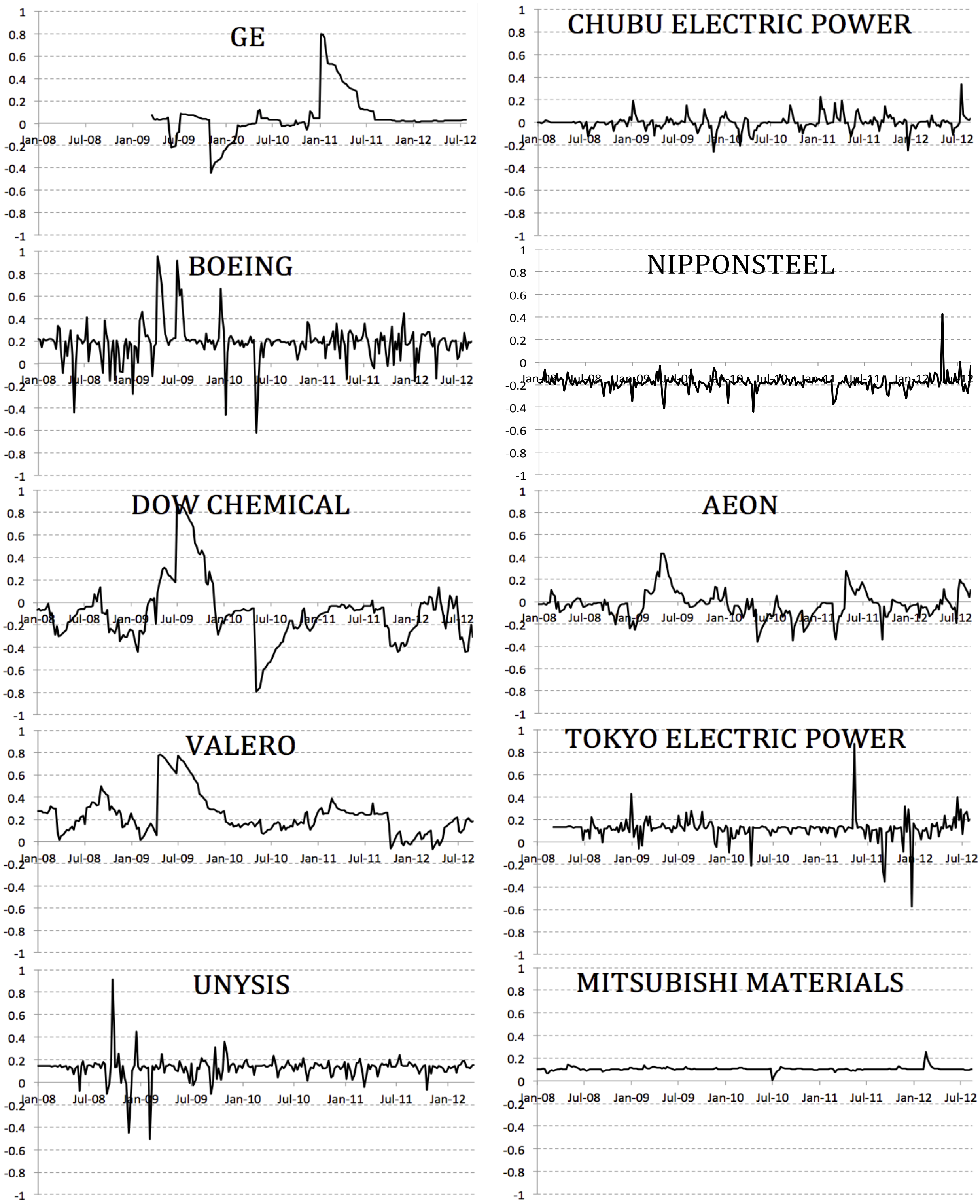


Figure 3: Estimated dynamic correlations between BAS and Quotes

market, irrespective of credit ratings, there exists a large variability across the names. This big swing of correlation coefficients indicate that it is unlikely that there exists a stable (over time) linear relationship between the two variables.

2.2.2 BAS-Quotes

Next, from Figure 3, we see that the variability in correlation coefficients (between Quote numbers and BAS) is not as large as in the previous pair. This may explain weaker causality relationship between the two variables. (Recall the results in Table 6.) But there are sharp pikes occasionally. See for example Boeing (A): the correlation coefficient is very low around 0.2 in most of the times but it jumped up to 0.8 from time to time.

For the purpose of predicting BAS, we believe that short-run analytical tool is of more practical use. As experienced in the recent financial crisis, liquidity crisis would break out all of sudden. The cost to hedge credit risks could surge in the short run. Thus, it would be helpful if one could foresee near-future BAS levels when trying to load and/or unload credit risks. From this point of view, although the causality relationship is stronger (at least in the long run) between Bid price and BAS, we shall concentrate on the Quote numbers to explain BAS. The occasionally observed high level of correlation between Quote numbers and BAS may provide us with a useful tool to predict liquidity levels in the short run.

3 The Hawkes Process

3.1 Specification

We build models to estimate and predict bid-ask spreads (BAS) in two steps: Let M be a standard Poisson random measure on $\mathbb{R}_+ \times \mathbb{R}_+$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ where $(\mathcal{F}_t)_{t\geq 0}$ is the augmented filtration generated by M. Let B be a standard Browinian motion independent of M. Define λ and N by setting $N_0 = 0$, $\lambda_0 = a$ and

$$\lambda_t := ae^{-ct} + \int_{(0,t)} be^{-c(t-s)} \mathrm{d}N_s, \quad t > 0, \tag{3.1}$$

$$N_t := \int_{[0,t]\times\mathbb{R}_+} M(\mathrm{d}s, \mathrm{d}z) 1_{(0,\lambda_s]}(z), \quad t > 0, \tag{3.2}$$

where $a, b, c \in (0, \infty)$ are constants. Refer this formulation to Çinlar[4]. It is called *self-exciting* (or the Hawkes process) in the sense that λ_t is affected by the path of N over $(0, t)$. The impact of N decays exponentially over time with rate c and the reversion level is a. The sensitivity parameter b takes care of the impact on λ when N increases. In our context, N is for the number of quotes and λ is the intensity of arrivals of these quotes. Moreover, we model the evolution of BAS, $X = \{X_t, t \geq 0\}$, by the Brownian subordination of N:

$$X_t := X_0 \exp\left(\left(\alpha - \frac{\beta^2}{2}\right) N_t + \beta B_{N_t}\right). \tag{3.3}$$

Since N is an increasing Lévy process and B is a Brownian motion in $\mathbb{R}$ independent of N, X is also a Lévy process (Theorem VII.6.2.[4]). The simple idea is the following: In Section 2.2 we observe when there are enough numbers of quotes, the correlation between BAS and quote numbers becomes instantaneously higher and the changes in BAS are not uniform in the real time. Accordingly, this time-change seems natural. That is, BAS changes when there arrives a quote. Figure 4 show some sample paths of the intensity process λ_t and the corresponding counting process N_t. Figure 5 are some sample paths of the intensity process λ_t with the corresponding Brownian subordination X_t.

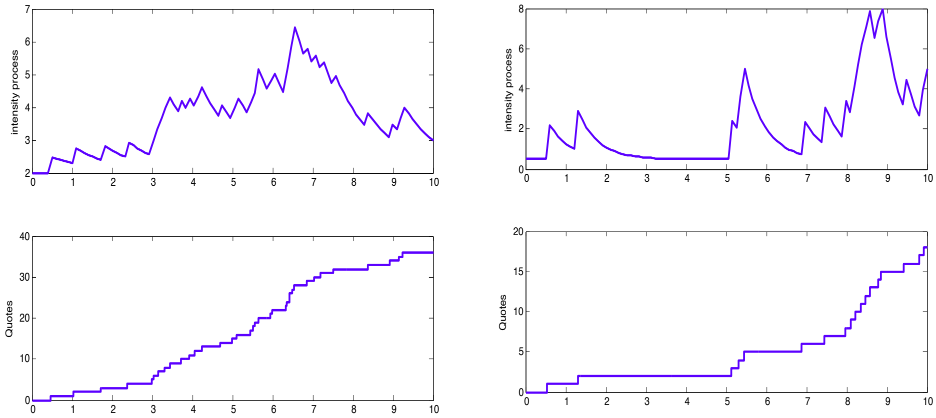


Figure 4: Hawkes process

Top: intensity process, Bottom: accumulating Quotes; left: t=10, a=2.0, b=0.5, c=1.0, right: t=10, a=0.5, b=2.0, c=2.0

Our estimation of the parameters in (3.1) is based on Ozaki [14]. Let $t_1, t_2, \ldots, t_n$ be the times of occurrence, the log-likelihood function of N_t is written

$$\log L(t_1, t_2, \ldots, t_n | \theta) = -\int_0^T \lambda(t|\theta) \mathrm{d}t + \int_0^T \log \lambda(t|\theta) \mathrm{d}N_t$$

where $[0, T]$ is the observation period. In the specification of (3.1), it becomes

$$\log L(t_1, t_2, \ldots, t_n | \theta) = -at_n + \sum_{i=1}^{n} \frac{b}{c}(e^{-c(t_n - t_i)} - 1) + \sum_{i=1}^{n} \log\left(a + \sum_{j}^{i} e^{-c(t_i - t_j)}\right). \tag{3.4}$$

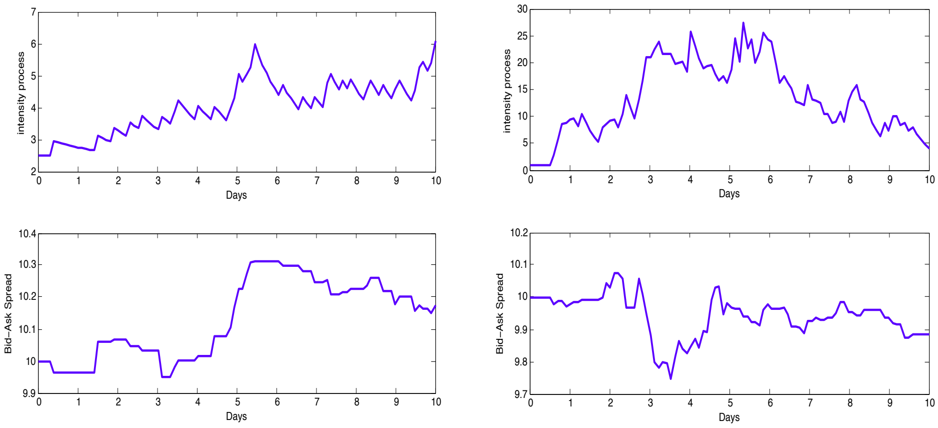


Figure 5: Brownian Subordination

Top: intensity process, bottom: Bid-Ask Spread ($\alpha = 0.0005$, $\beta = 0.003$), left: t=10, a=2.5, b=0.5, c=1.0, right: t=10, a=0.5, b=2.0, c=1.5

For the parameters α and β in (3.3), we use the standard formula for geometric Brownian motion:

$$\log X_{t+\Delta t} - \log X_t = \left(\alpha - \frac{1}{2}\sigma^2\right) \Delta N_t + \beta^2 \Delta N_t.$$

As for simulation, we use Algorithm 2 in Ogata[12] and we reproduce it for the record. By assuming that the minimum value of the intensity function is μ, the jump size at each point is not larger than B, and the Λ_i are values, of the piecewise constant function such that $\lambda(t|t_1, \cdots, t_n) \leq \Lambda_i$ for $t_n \leq s_i \leq t < s_{i+1} \leq t_{n+1}$.

1) Set $\Lambda_0 = \mu$ and $s_0 = 0$.

2) Generate a random variable U_0 uniform in $(0, 1)$ and put $u_0 = -\log(U_0/\Lambda_0)$.

3) If $u_0 \leq T$, then put $t_1 = u_0$. Otherwise stop.

4) Set $i = j = k = 0$ and $n = 1$.

5) Set k equal to $k + 1$ and put $\Lambda_k = \lambda(t_n|t_1, \ldots, t_{n-1}) + B$.

6) Set j equal to $j + 1$ and generate U_j.

7) Set i equal to $i + 1$ and put $u_i = -\log(U_j/\Lambda_k)$.

8) Put $s_i = s_{i-1} + u_i$. If $s_i > T$, stop.

9) Set $j = j + 1$ and generate U_j.

10) If $U_j \leq \lambda(s_i|t_1, \cdots, t_{n-1})/\Lambda_k$, set n equal to $n + 1$, put $t_n = s_i$ and go to step 5. Otherwise go on.

11) Set k equal to $k + 1$, put $\Lambda_k = \lambda(t_n|t_1, \ldots, t_{n-1})$ and go to step 6.

Just as a preliminary test, we use the samples of one month data June 2012 $\sim$ July 2012 and estimate the parameters a, b and c. (See Table 2.) Note that when feeding data into (3.4), we assume that quotes in one day are uniformly distributed within the day. By using the estimates, we simulate quote numbers and compare a sample path of N_t with the actual quotes. See Figure 6. The result is encouraging and we proceed with some full-fledged estimations in the next subsection.

Table 2: Parameter Estimation for the Preliminary Testing

		Quotes			Bid-Ask Spread	
	log-likelihood	$\hat{a}$	$\hat{b}$	$\hat{c}$	$\hat{\alpha}$	$\hat{\beta}$
Toyota Motor	722.229	5.706171	1.506171	2.13881	-0.0004221298	0.005755804
MetLife	3165.603	6.230013	2.830023	3.191574	0.0006556054	0.0034706

3.2 Prediction

Let us move on to predictions of BAS via the Hawkes model. We shall conduct two types of performance tests.

3.2.1 First Test

We take 5 year CDS contracts and prepare three sets of in-sample period (two-and-half years) of Quotes and BAS:

(a) January 2008 to June 2010

(b) January 2009 to June 2011

(c) January 2010 to June 2012

with the respective out-of-sample period (with the length of one month) July 2010, July 2011, and July 2012. The first two periods (a) and (b) include the financial crisis within their range. By using these in-sample period data, we estimate the model parameters in (3.1) and (3.3). Then we conduct Monte

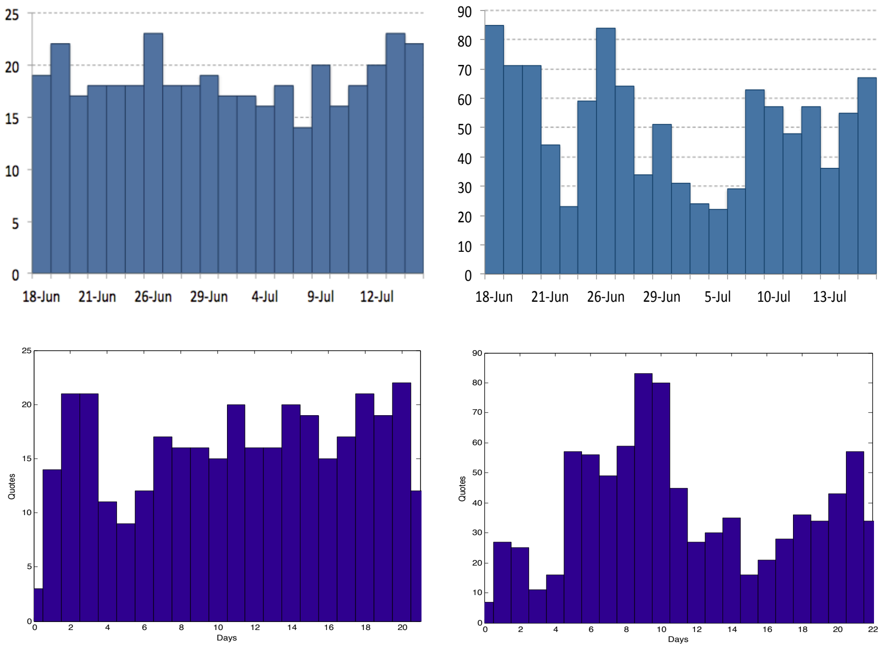


Figure 6: Comparison of observed (top) and simulated (bottom) quote numbers
Left: Toyota Motor, right: MetLife

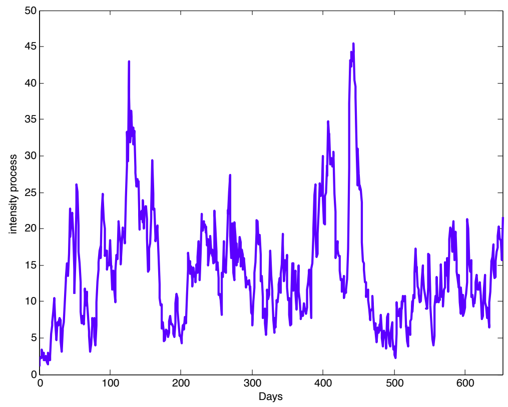


Figure 7: Intensity simulation
T=650, a=1.105, b=0.7034, c=0.7504

Carlo simulations to predict the BAS's for the out-of-sample period (for the next one month), which are compared with the actual data.

In this experiment, note that we have to take into account the following fact: If we use the Quote data of Toyota from Jan 2010 to June 2012 (two and half years), the parameters in (3.1) are estimated as $(\hat{a}, \hat{b}, \hat{c}) = (1.105, 0.7034, 0.7504)$. A typical sample path of the Monte Carlo simulations of the intensity process is shown in Figure 7. It starts with $\hat{a} = \lambda_0 = 1.105$, but $(\lambda_t)_{t>0}$ spends throughout the period above the level 1.105 and at this end of this period (two and half years), λ_T is above 20. Hence it is by no means appropriate to use $\hat{a} = 1.105$ as the *initial* level of λ for the out-of-sample period that follows. In view of this, we did the following two-step estimation:

(1) Use the whole two-and-half year sample data to estimate $(\hat{a}, \hat{b}, \hat{c})$.

(2) Use only the final one month period, re-estimate a, while fixing $(\hat{b}, \hat{c})$ at the values in step (1). The result is, say, $(\hat{a'}, \hat{b}, \hat{c})$.

(3) Simulate $(N = 10,000)$, with $(\hat{a'}, \hat{b}, \hat{c})$, the intensity $(\lambda_t)_{t>0}$ for a one-month period.

This procedure is justified also by our observation that the values of (b, c) are not so much varying within the two-and-half year periods. These parameters are rather characteristic of CDS reference (=company) names. The results are shown in Table 7. The reference names are Dell, AT&T, AK Steel, Belo, Valero from the U.S. market and Toyota, Sumitomo, Softbank, Acom, and Kintetsu from Japan. We compare the actual data and simulated data in Table 8. The first (without counting the column where we put company names) column shows the observed average number of daily Quotes. The second column is the 95 % confidence interval of the simulated mean. The third column is the observed average BAS in the respective one-month period, and the fourth column is the 95% confidence interval of the mean of simulated daily BAS's. The confidence intervals obtained by the Hawkes model for BAS are very narrow and generally well capture the observed BAS average within their ranges. In the periods (b) and (c), it is very rare that the estimated intervals differ from the observed spreads by more than 3 basis points. The exceptions are AK Steel and Kintetsu in period (c). Considering the facts that the period (b) contains the financial crisis in its range and the names contain both high and low credit ratings, the Hawkes model consistently well performs. Turning to the period (a), we see that the model is good in the U.S. market. But in this period, the model fails to capture the BAS for low credit names in the Japanese market (Softbank, Acom, and Kintetsu). Based on the regression results in Section 1.1, this is due to the underdeveloped state of the CDS market in Japan, especially low transaction volumes in lower credit contracts. The sixth column is the standard deviations of the observed BAS's during the

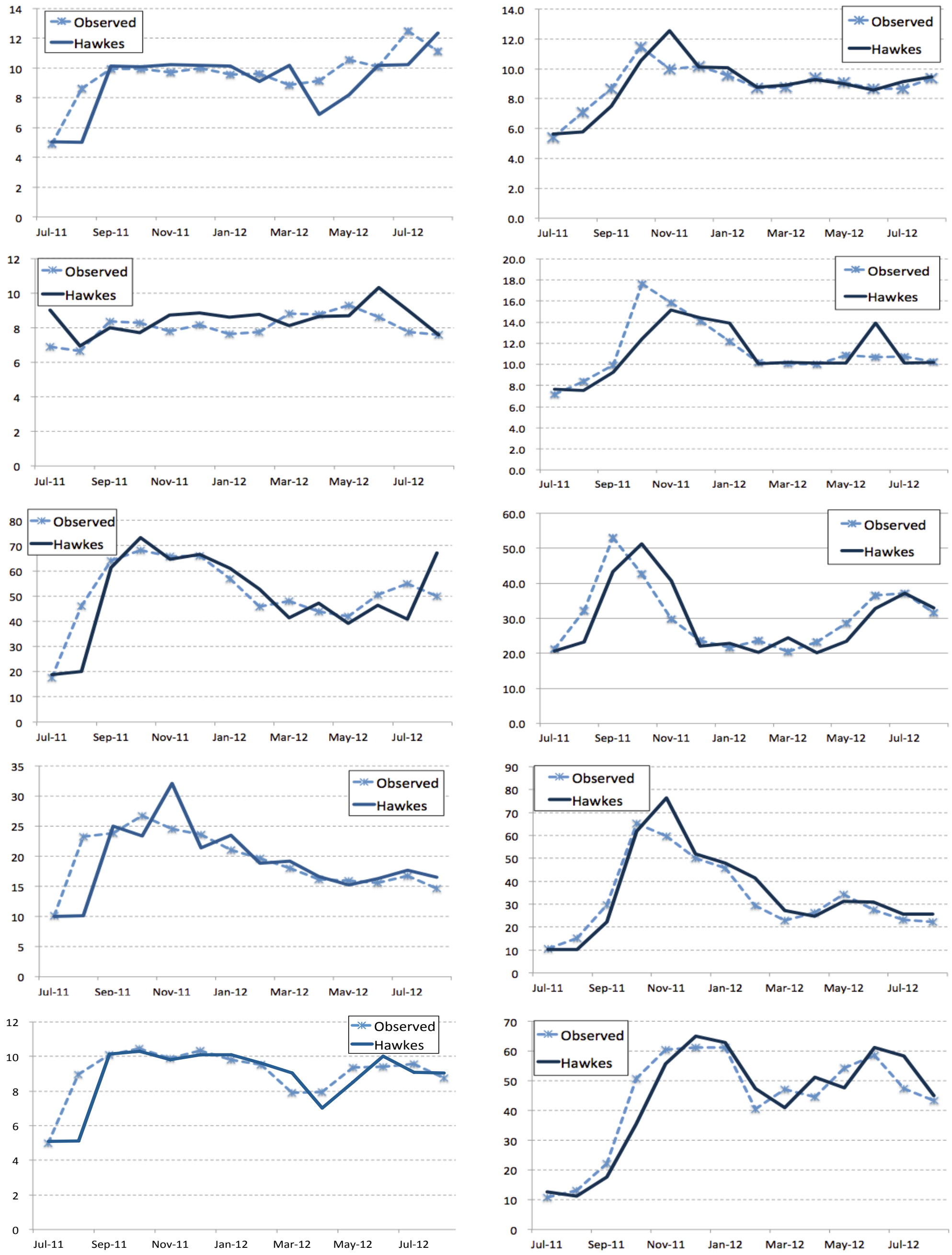


Figure 8: Comparison of 14 month bid-ask spreads (actual vs predicted)
Left: U.S.(from the top, Dell, AT&T, AK STEEL, BELO, VALERO), Right: Japan(from the top, TOYOTA, SUMITOMO, ACOM, SOFTBANK, KINTETSU)

one month (out-of-sample) periods. For the seventh column, we compute standard deviations of each predicted sample path of BAS's and take the average over $N = 10,000$ simulations. The left end values of the confidence intervals are in general greater than the observed standard deviations. The fifth and eight columns show the 95% confidence interval obtained from ARMA-GARCH (1, 1). As is clearly seen, these values are by far worse than the results from the Hawkes process.

3.2.2 Second Test:

Again, we take 5 year CDS contracts. The next prediction experiment is done in the following way:

(a) Take fourteen sets of two-and-half year in-sample periods; (1) Jan 2009 $\sim$ June 2011, (2) Feb 2009 $\sim$ July 2011, $\cdots$, and (14) Feb 2010 $\sim$ July 2012, so that each adjacent set is lagged by one month.

(b) For each set, we estimate the model parameters and use these values to predict the out-of-sample periods; (1) July 2011, (2) August 2011, $\cdots$, and (14) August 2012, respectively.

(c) These predicted BAS are compared with the actual data for the fourteen months from July 2011 to August 2012. We did this procedure to the same ten companies as in the *First Test*.

We plot the BAS numbers (actual vs. predicted) in Figure 8. For the full results, see Tables 9 and 10 that show the estimated parameters for the two countries, and Tables 11 through 15 include actual and predicted Quotes and BAS numbers. Of course not perfect, the simulated prediction fits the actual observation pretty well. At least, the up and down trends are nicely tracked in all of the cases: both fairly stable BAS's and volatile ones. From what we observe, the Hawkes process-based prediction performs well even though there are difference between the U.S. and Japanese markets, and between higher and lower rating grade companies.

4 Conclusion

We have done static and dynamic analysis for both the U.S and Japanese CDS markets focusing on the bid-ask spreads. We indicate similarities and differences between the two markets and also between the high credit and low credit ratings in the above sections. In particular, in the dynamic part, we treat both long run (i.e., DCC GARCH) and short run analysis (i.e., the Hawkes process fitting). The Hawkes process based model for bid-ask spreads fits the data well. We believe that this model can be a useful tool for liquidity risk management as well. Investors with a large portfolio of credit risk exposures may be able to estimate transaction costs and, more importantly, foresee possible surge of bid-ask spreads ahead of time. The latter could lead to a significant amount of cost savings.

In doing prediction, one could use the most recent actual values available in the market for a in (3.1), instead of using the estimated values $\hat{a}'$ we did above. This parameter is very important because the initial level of the intensity process λ, and may improve the accuracy of predictions.

A Tables

Table 3: Adjusted R^2 in Regression and partial F-test

	Rating	Regres. (1.1)		Regres. (1.2)		F-statistics	
	S& P	one-year	five-year	one-year	five-year	one-year	five-year
GENERAL ELECTRIC	AA+	-0.0041	-0.0141	0.4837	0.4309	0.8926	0.4893
WALMART STORES	AA	0.0736	0.1378	0.0614	-0.0057	1.264	1.067
BOEING	A	0.0828	0.1583	0.0359	0.0610	1.450	0.5942
McDONALDS	A	0.0236	0.1283	0.0808	-0.00631	0.8800	1.205
METLIFE	A-	0.4153	0.4813	0.2875	0.4852	2.633*	4.740**
DELL	A-	0.0050	0.1734	-0.0056	-0.0055	3.504**	2.201*
AT & T	A-	0.0347	0.0202	0.0040	0.3993	1.112	4.301**
AMERICAN ELECTRIC POWER	BBB	0.0493	0.0096	-0.0056	0.012	1.014	1.108
VALERO ENERGY CORP	BBB	0.1416	0.0656	-0.0047	-0.0060	0.7455	0.9623
DOW CHEMICAL	BBB	0.2116	0.2845	0.0891	0.0219	0.8108	1.3201
average in Investment Grade		0.1033	0.1532	0.1026	0.1387		
INTERNATIONAL LEASE FINANCE	BBB-	0.2448	0.3183	0.0413	0.2742	5.071**	13.67**
FORD MOTOR	BB+	0.1717	0.1956	0.2289	0.5616	1.036	3.213**
AK STEEL	BB-	0.0374	0.1900	0.0280	0.0065	0.5910	0.8333
BELO CORP	BB-	0.0249	0.0848	0.0244	0.1496	5.092**	1.514
FOREST OIL	BB-	0.1009	0.2579	0.0459	0.1017	2.2331*	2.414*
UNISYS	BB-	0.2013	0.2590	-0.0055	0.1197	8.763**	17.30**
IRON MOUNTAIN	BB-	0.0526	0.0446	-0.0044	0.0728	1.960	1.559
CHESAPEAKE ENERGY	BB-	0.0750	0.2905	-0.0001	0.0892	2.548*	7.405**
ALLY FINANCIAL	B+	0.6291	0.6121	0.1810	0.2054	4.611**	9.877**
MGIC INVESTMENT	CCC+	0.0452	0.1906	0.0389	0.2472	14.63**	1.830
average in High Yield		0.1583	0.2443	0.0578	0.1887		
average in U.S.		0.1308	0.1988	0.0802	0.1637		

	Rating	Regres. (1.1)		Regres. (1.2)		F-statistics	
	R& I	one-year	five-year	one-year	five-year	one-year	five-year
TOYOTA MOTOR	AAA	0.0184	0.0586	0.0114	0.0067	1.120	1.844
CHUBU ELECTRIC POWER	AA	0.0044	-0.0030	-0.0028	0.0291	0.9378	2.163*
NIPPON STEEL	AA	0.0689	0.1650	-0.0031	-0.0047	0.7972	0.8567
ASAHI GROUP	AA-	-0.0001	0.0037	-0.0031	-0.0047	0.5756	0.3447
NOMURA HOLDINGS	AA-	-0.0050	-0.0133	0.0033	-0.0059	0.9868	1.311
SUMITOMO CORP	AA-	0.0127	0.0142	0.0466	0.0687	3.412**	3.670**
HITACHI	AA-	0.0524	0.1647	0.0735	0.1605	1.807	2.367*
SOFTBANK	A	-0.0046	0.0071	-0.0014	0.1164	2.862**	2.022
ACOM	A	0.0850	0.1114	0.0263	0.0157	1.394	1.574
AEON	A	0.0152	0.0420	0.0179	0.088	2.324*	3.942**
average in investmant grade		0.0247	0.0550	0.0188	0.047		
MAZDA MOTOR	BBB+	-0.0039	-0.0129	0.1507	0.1403	1.266	1.658
MITSUBISHI MATERIALS	BBB+	-0.0125	-0.0036	0.0959	-0.0062	1.298	1.061
KINTETSU	BBB+	0.0494	0.0876	0.1153	0.0103	0.6445	0.3521
SAPPORO HOLDINGS	BBB+	-0.0128	-0.0097	0.2300	0.3034	2.100	1.012
IHI	BBB+	-0.0133	-0.0155	-0.0049	0.0208	1.387	1.873
MAEDA	BBB+	-0.0248	0.0117	-0.0067	0.1166	0.3202	0.3023
ANRITSU	BBB+	0.0271	-0.0324	0.9485	0.7406	3.127**	1.524
SHINSEI BANK	BBB	0.0002	-0.0013	0.2537	0.0467	0.6448	0.6355
PIONEER	BBB	0.051	0.2000	-0.0049	0.0208	0.4197	1.479
AIFUL	CCC	0.0112	0.0189	0.1708	0.3488	0.5414	0.5314
average in High Yield		0.0018	0.0243	0.1741	0.1716		
average in Japan		0.0132	0.0397	0.0965	0.1093		

p-value less than **:1 percent, *:5 percent

Table 4: Coefficients to regress Bid-Ask Spreads in one-year contract CDS

	Δ(Bid)	Δ(Interest)	Δ(VI)	Δ(Volume)	Δ(StockPrice)	Δ(StockSigma)	Δ(StockVol)	Δ(Crisis)
GE	0.2907**	1.4353	-0.0523	-1.2417	10.1862	11.9972	1.1781	
WALMART	-0.2075**	0.3970	0.1183*	0.7001	5.7987	-74.0718	0.5392	-0.0041
BOEING	0.0945**	0.4920	-0.0594	-0.6176	15.8032	50.7611	-0.8783	0.0916
MCDONALDS	-0.2545**	-1.2862	0.0246	-1.0882	-7.9366	0.1341	0.8426	0.0544
METLIFE	0.2064**	-6.5181	-0.1835	-6.4302	118.0829	191.4951	9.9764	0.5749
DELL	0.0225	-0.3574	-0.1762**	0.4616	-24.7058*	-86.5426**	3.3039**	-0.0636
AT& T	-0.0667*	0.2654	0.0742	0.5945	19.1169	8.7033	1.8140	0.0819
AMERICAN ELEC	0.0058	-1.4441	-0.0414	-0.4360	-4.9118	41.7283	0.0246	0.0354
VALERO	-0.0025	-2.3010	-0.0199	-0.2877	16.0494	1.7587	0.7588	0.1094
DOW CHEM	0.0709**	-2.2646	-0.0547	-0.0598	8.6793	28.0496	1.6709	0.0262
FORD	0.1271***	60.4911	-0.4105	-9.4144	-182.8653	111.2600	16.0730	0.8918
AK STEEL	0.0659**	3.9242	0.0223	-4.1301	-44.3807	-111.5615	3.1360	-0.5274
BELO	0.0571**	6.5147	2.0444**	17.6603	-93.0524	-373.1725**	-14.3063	0.6033
FOREST OIL	0.0949**	-12.7739	-0.8435**	-2.2583	-36.3112	-105.9673	0.5321	-0.0171
UNISYS	-0.0575*	53.3839	-5.9294*	121.096*	-648.0591**	2431.2284**	-153.4435**	0.70194
IRON MOUNT	-0.000765	-5.2244*	0.1296	2.6758	-1.3485	7.3423	-1.2660	-0.1488
CHESAPEAKE	0.0874	-13.5363	-2.4360**	18.6688	34.2353	292.7386	5.8817	0.2696
ALLY FIN	0.121**	55.520	0.277	65.886	249.358	1424.196**	10.371	-1.233
INTER LEASE	0.0431	-45.2685	-1.3462	8.5344	-200.9290**	258.4313*	-9.8886	-1.7914
MGIC	-0.1772**	0.3973	-0.8202	-57.8877*	-443.8731**	854.4722**	1.2105	-2.9807

	Δ(Bid)	Δ(Interest)	Δ(VI)	Δ(Volume)	Δ(StockPrice)	Δ(StockSigma)	Δ(StockVol)	Δ(Crisis)
TOYOTA	0.0750*	-1.7605	0.0215	-0.6001	-30.2484*	-19.3537	0.6859	-0.1581
CHUBU	0.0483	-0.4397	0.0960*	-0.6300	13.2446	-34.4850	0.6574	0.1404
NIPPON STEEL	0.0398	0.0909	0.1136	-3.2692	5.6861	34.0362	-0.2207	-0.0197
ASAHI	-0.1743**	0.1162	-3.6418	-0.8729	-12.7071	-57.8543	0.8834	-0.0608
NOMURA	0.0535	1.29484	-0.0089	-4.9527	-72.4630*	-59.0562	-1.0178	-0.4256
SUMITOMO	0.2001**	-9.5908	-0.0704	-4.9918	90.6216**	40.8240	5.0567	-0.7049
HITACHI	0.2784**	-7.6758	0.1962	2.9388	24.4244	61.2449	-5.4337*	0.0014
SOFTBANK	0.0653	27.1565	-0.4313	-69.9410**	-30.2923	-26.3252	-10.9522	-0.7359
ACOM	-0.2270*	0.8249	1.6660	63.1870	20.3758	342.1862	-39.6392	-0.4889
AEON	-0.1710**	22.1799	-0.4404	-2.9913	-154.3648**	-230.6126	9.9578	-0.0267
MAZDA	0.3077**	6.9161	-0.4788	8.3572	1.0218	-7.8888	3.7957	-0.0118
MATERIALS	-0.2066**	-2.4028	0.1223	1.7258	-15.3096	-67.9243	1.6973	1.1109
KINTETSU	-0.3615**	-12.0857	0.0533	-8.5618	-7.2398	-121.4281	4.1523	-0.2595
SAPPORO	0.4274**	6.7441	-0.5101	-7.5348	-117.4324*	255.8141	-0.5101	
IHI	-0.0030	-17.8202	-0.5927	-48.1890*	6.6656	180.2065	22.9154	-0.3996
MAEDA	-0.0281	-10.6764	-0.0324	2.7386	-2.0083	-47.6794	-0.9447	
ANRITSU	0.1504**	-0.0388	0.0004	0.0574	-0.1979	-1.9310**	-0.0332	
SHINSEI	-0.4533**	-9.1703	-1.2079	5.0721	-226.8239	-79.9928	-13.5235	-4.1858
PIONEER	0.0119	-0.5142	0.0710	-6.1342	0.0738	-6.0303	1.8319	0.0658
AIFUL	0.1181**	94.4605	4.6687	-178.1841	309.5854	505.0846	41.0710	19.0328

p-value less than **:1 percent, *:5 percent

Table 5: Coefficients of VAR and F statistics in the Granger causality test

	b_{21}		b_{22}		$\Delta(BAS)_{t-1} \rightarrow \Delta(Bid)_t$	
	one-year	five-year	one-year	five-year	one-year	five-year
GE	0.1112	0.0013	-0.6327	-0.3711	3.9086*	0.0281
WALMART	0.1027	0.0420	-0.3586	-0.4183	6.5872*	0.0284
BOEING	0.0973	0.0393	-0.4157	-0.3270	0.0443	1.6247
MCDONALDS	0.0821	0.0122	-0.2437	-0.5034	4.3776*	0.0505
METLIFE	0.1359	0.0077	-0.2222	-0.1810	0.6520	0.0021
DELL	0.1844	0.05668	-0.2214	-0.3842	13.2521**	13.2715**
AT& T	0.1388	-0.1948	-0.3243	-0.5985	2.3919	79.4863**
AMERICAN ELEC	0.0481	0.02366	-0.4518	-0.4895	2.5624	1.5244
VALERO	0.0266	0.0072	-0.4184	-0.4929	6.8563**	0.9108
DOW CHEM	0.0261	0.0152	-0.5347	-0.4633	2.7214	0.0932
FORD	-0.0348	0.0424	-0.2581	-0.4149	46.7527**	2.2168
AK STEEL	0.0375	0.0194	-0.4400	-0.5855	0.6098	0.0654
BELO CORP	0.0997	0.0241	-0.3302	-0.0891	32.7864**	4.4147*
FOREST OIL	0.0063	0.0243	-0.3302	-0.3853	8.7364**	0.2129
UNISYS	0.0342	0.0242	-0.2074	-0.0296	10.3175**	2.162
IRON MOUNT	-0.0336	-0.0111	-0.3493	-0.5323	1.1223	6.7821**
CHESAPEAKE	0.2038	0.0645	-0.4258	-0.3973	5.3495*	0.0777
ALLY FIN	0.0907	0.2201	-0.2989	-0.6264	24.0233**	0.695
INTER LEASE	0.0221	0.0112	-0.4850	-0.4874	1.5611	6.3805*
MGIC	-0.0077	0.0320	-0.3865	-0.4915	1.6269	13.1742**

	b_{21}		b_{22}		$\Delta(BAS)_{t-1} \rightarrow \Delta(Bid)_t$	
	one-year	five-year	one-year	five-year	one-year	five-year
TOYOTA	0.2656	0.0985	-0.3140	-0.3607	13.2953**	8.0016**
CHUBU ELEC	-0.0087	0.0519	-0.0764	-0.1593	3.8622*	1.3519
NIPPON STEEL	0.2916	0.1413	-0.3812	-0.3846	0.094	2.1191
ASAHI	0.2981	0.0983	-0.1464	-0.2458	20.413**	12.8373**
NOMURA	0.0013	0.0013	-0.2645	-0.3155	0.6699	0.4889
SUMITOMO	0.1218	0.0668	-0.3071	-0.3627	0.0183	3.9927*
HITACHI	0.2713	0.1992	-0.3545	-0.3038	1.9351	11.9009**
SOFTBANK	0.1122	0.1495	-0.4015	-0.4195	2.6972	0.5928
ACOM	0.0253	0.0164	-0.3948	-0.3982	34.4122**	5.4584*
AEON	0.2897	0.0972	-0.1583	-0.2863	55.3525**	43.4629**
MAZDA	0.0519	0.0293	-0.1650	-0.1877	17.302**	6.8298**
MATERIALS	0.1888	0.0338	-0.3032	-0.3713	7.2391**	0.0021
KINTETSU	0.3480	0.2253	-0.3133	-0.3750	7.0442**	0.0974
SAPPORO	-0.0815	-0.0057	0.1466	-0.0046	13.1345**	23.5656**
SHINSEI	0.00327	0.0238	-0.0980	-0.1220	1.5847	0.0025
PIONEER	0.0345	0.0328	-0.2008	-0.3238	0.0034	0.3243
AIFUL	-0.0823	-0.0734	-0.1692	-0.1296	2.9913	1.1188
IHI	0.1127	0.0314	-0.0173	-0.0565	2.1031	1.1289
MAEDA	0.0190	-0.0059	-0.0120	-0.0586	0.0032	3.1155
ANRITSU	-0.0275	0.0781	-0.0063	-0.1524	0	0.0365

p-value less than **:1 percent, *:5 percent

Table 6: Granger causality test between Bid and Quotes, BAS and Quotes

	$\Delta(Bid)_{t-1} \to \Delta(Quotes)_t$	$\Delta(Quotes)_{t-1} \to \Delta(Bid)_t$	$\Delta(BAS)_{t-1} \to \Delta(Quotes)_t$	$\Delta(Quotes)_{t-1} \to \Delta(BAS)_t$
GE	0.0126	0.1467	0.488	0.0018
WALMART	0.3211	0.3495	1.3608	1.6334
BOEING	0.6142	0.5963	2.8584	4.1077*
MCDONALDS	0.7744	0.0283	0.0082	0.4433
METLIFE	9.1977**	0.9902	1.8125	4.1901*
DELL	0.0888	2.6586	1.1392	26.8677**
AT& T	0.0316	0.0198	0.7673	0.5893
AMERICAN ELEC	11.0412**	0.1619	1.8172	2.1367
VALERO	0.8376	0.0053	6.7073**	0.3051
DOW CHEM	2.2826	0.1150	0.9203	2.8238
FORD	0.0565	14.2146**	0.1062	0.0788
AK STEEL	0.9162	3.8678*	0.433	0.3012
BELO CORP	1.3848	0.0719	0.2586	2.0608
FOREST OIL	2.2936	0.1359	1.6904	0.9324
UNISYS	0.7487	0.4223	0.2903	1.3612
IRON MOUNT	1.1246	1.3863	69.8123**	1.8085
CHESAPEAKE	9.2459**	0.0202	10.7715**	3.9106*
ALLY FIN	0.0766	0.1347	0.0125	0.0495
ARAMARK	0	0.0775	0.0048	0.4188
SPRINGLEAF	0.0424	0.1672	0.0515	0.952

	$\Delta(Bid)_{t-1} \to \Delta(Quotes)_t$	$\Delta(Quotes)_{t-1} \to \Delta(Bid)_t$	$\Delta(BAS)_{t-1} \to \Delta(Quotes)_t$	$\Delta(Quotes)_{t-1} \to \Delta(BAS)_t$
TOYOTA	0.2123	0.9189	0.0206	0.3146
CHUBU ELEC	2.1346	0.2582	0.3988	0.5898
NIPPON STEEL	1.4048	0.5576	0.4671	0.1592
NOMURA	0.9771	0.2091	0.2473	0.3415
SUMITOMO	0.0004	0.0221	0	0.0331
HITACHI	0.0805	0.0294	0.9203	2.4511
SOFTBANK	5.578*	0.4172	0.5381	0.0765
ACOM	0.7454	0.0847	0.0003	2.0202
AEON	0.0548	1.7679	0.0663	0.2014
SHARP	2.8663	1.0018	0.5245	1.5966
TOKELP	0.4357	0.0143	1.1331	0.2273
MAZDA	1.2618	2.8038	0.968	6.2024*
MATERIALS	1.134	0.0593	1.6805	2.4058
KINTETSU	1.3664	0.0197	0.6691	6.0978*
SAPPORO	0.001	0.0076	0.2227	0.9759
SOJITZ	0.0016	1.2445	0.9577	1.0536
SHINSEI	0.0242	3.0059	0.3434	0.094
PIONEER	1.4086	14.2741**	34.2881**	8.9154**
AIFUL	1.2228	0.0736	0.1074	0

p-value less than **:1 percent, *:5 percent

Table 7: Estimated Parameters from the Hawkes Model

July-2010	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
DELL	0.00006651	0.01076	5.3136	1.9640	2.0275
AT&T	0.0004781	0.02942	3.0525	0.73559	0.77562
AK STEEL	0.00001813	0.005699	1.9697	4.4200	4.4961
BELO	0.00001244	0.005040	4.6798	5.4263	5.4675
VALERO	0.00007151	0.01100	3.5399	3.1716	3.2918
TOYOTA	0.0002054	0.01954	4.2457	0.58286	0.62786
SUMITOMO	0.0003156	0.02314	2.4338	0.59184	0.66534
SOFTBANK	0.0007965	0.03557	1.2520	0.70781	0.74581
ACOM	0.0005500	0.02761	2.0678	0.57667	0.63167
KINTETSU	0.0007635	0.03358	2.8860	0.28772	0.30472

July-2011	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
DELL	0.00001504	0.007841	5.6042	2.3983	2.4643
AT&T	0.0002281	0.02174	3.3623	1.0487	1.1244
AK STEEL	0.000004735	0.005419	4.3726	4.5364	4.6039
BELO	0.00000003761	0.004458	5.2618	4.8533	4.9274
VALERO	0.00002346	0.009111	5.2234	3.3609	3.4746
TOYOTA	0.00001659	0.01356	2.6224	0.74000	0.78350
SUMITOMO	0.00005132	0.01686	2.8330	0.68333	0.73233
SOFTBANK	0.0003955	0.03520	1.6353	0.59403	0.64523
ACOM	0.0002321	0.02443	3.1280	0.64436	0.70577
KINTETSU	0.00005990	0.01726	3.4686	0.50163	0.55862

July-2012	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
DELL	0.00002012	0.005093	5.8658	2.5437	2.6073
AT&T	0.0001426	0.01595	2.5936	1.1945	1.2901
AK STEEL	0.00006131	0.009857	4.9799	2.1003	2.1417
BELO	0.00001767	0.006032	4.1613	2.4111	2.4738
VALERO	0.00001138	0.005074	7.8070	2.2409	2.4085
TOYOTA	0.00007224	0.01202	3.8004	0.70337	0.75037
SUMITOMO	0.00006464	0.01380	3.3897	0.69198	0.75398
SOFTBANK	0.0002347	0.02638	2.9605	0.41663	0.46977
ACOM	0.0001325	0.01707	2.4390	0.64387	0.72749
KINTETSU	0.0001388	0.01364	3.9440	0.55844	0.62944

Table 8: Confidence Interval

July-2010	Quotes		Average			Standard Deviation		
	Observed	Hawkes	Observed	Hawkes	ARMA	Observed	Hawkes	GARCH
DELL	759	[959.0, 1016.6]	6.631	[8.010, 8.089]	[3.082, 14.14]	1.086	[0.5467, 0.5841]	[4.873, 18.95]
AT&T	305	[473.0, 496.8]	6.706	[8.119, 8.463]	[0.673, 10.10]	0.832	[2.112, 2.363]	[4.051, 20.10]
AK STEEL	730	[1370.3, 1426.3]	32.13	[31.35, 31.48]	[13.06, 34.78]	3.975	[2.489, 2.559]	[17.51, 43.35]
BELO	1431	[4143.3, 4496.2]	13.33	[12.47, 12.67]	[12.67, 12.91]	1.696	[1.582, 1.709]	[505.1, 2195.1]
VALERO	884	[1396.7, 1506.9]	10.842	[10.312, 10.62]	[-1197300, 409600]	1.457	[1.721, 1.953]	[-325700, 4594300]
TOYOTA	405	[538.8, 545.2]	7.361	[6.592, 6.650]	[-26.01, 16.51]	0.8788	[1.197, 1.227]	[10.25, 39.20]
SUMITOMO	333	[275.8, 280.0]	8.620	[9.262, 9.333]	[-10.39, 22.41]	0.5574	[1.425, 1.460]	[32.62, 140.91]
SOFTBANK	148	[200.9, 205.6]	79.25	[123.8, 125.1]	[100.6, 112.8]	22.928	[25.26, 26.06]	[39.60, 95.48]
ACOM	198	[252.6, 256.8]	44.46	[79.35, 80.06]	[17.85, 197.9]	16.89	[14.20, 14.61]	[591.9, 1311]
KINTETSU	182	[246.01, 248.85]	27.39	[35.64, 36.01]	[67.47, 68.60]	2.588	[7.629, 7.844]	[10.22, 13.59]

July-2011	Quotes		Average			Standard Deviation		
	Observed	Hawkes	Observed	Hawkes	ARMA	Observed	Hawkes	GARCH
DELL	1206	[2046.2, 2172.0]	4.892	[4.953, 5.057]	[-308.4, 804.4]	0.116	[0.6997, 0.7508]	[41.07, 243.8]
AT&T	432	[2912.7, 2995.3]	6.912	[8.787, 9.081]	[-25.52, 75.85]	0.946	[1.826, 1.999]	[18.75, 174.6]
AK STEEL	714	[1832.8, 1896.0]	17.73	[18.77, 18.87]	[-165.4, 277.5]	1.965	[2.120, 2.170]	[22.45, 761.27]
BELO	1192	[1020.7, 1196.3]	10.11	[9.94, 10.02]	[0.698, 23.16]	0.341	[0.5053, 0.5505]	[14.72, 77.34]
VALERO	986	[2101.2, 2239.9]	4.992	[5.056, 5.195]	[-13333, 4568]	0.037	[0.8375, 0.9127]	[-11257, 85716]
TOYOTA	300	[385.6, 392.0]	5.417	[5.595, 5.623]	[-2261, 2171]	0.3462	[0.5841, 0.5961]	[-8940, 116610]
SUMITOMO	300	[376.5, 382.3]	7.160	[7.633, 7.680]	[7.729, 8.129]	0.2853	[0.9803, 1.0013]	[2.829, 4.160]
SOFTBANK	175	[195.1, 198.8]	10.62	[10.23, 10.33]	[3.822, 12.81]	0.8571	[1.978, 2.031]	[28.49, 54.26]
ACOM	282	[372.49, 377.73]	21.25	[20.64, 20.83]	[-8095, 13426]	1.391	[3.880, 3.981]	[48720, 189600]
KINTETSU	317	[343.7, 347.9]	10.81	[12.58, 12.66]	[19.16, 19.26]	0.6562	[1.594, 1.627]	[2.109, 2.175]

July-2012	Quotes		Average			Standard Deviation		
	Observed	Hawkes	Observed	Hawkes	ARMA	Observed	Hawkes	GARCH
DELL	1391	[2489.6, 2645.0]	12.51	[10.13, 10.28]	[5.974, 22.15]	2.854	[1.028, 1.101]	[1.790, 23.60]
AT&T	185	[500.6, 530.4]	7.765	[8.864, 9.067]	[5.634, 7.011]	0.562	[1.227, 1.331]	[0.437, 4.097]
AK STEEL	1366	[2027.2, 2066.4]	55.00	[40.52, 40.86]	[-168.1, 254.7]	10.92	[7.215, 7.414]	[18.30, 2267]
BELO	892	[1663.1, 1777.9]	16.74	[17.59, 17.85]	[-133.0, 378.7]	2.098	[1.663, 1.774]	[77.09, 695.3]
VALERO	1195	[1838.1, 1911.0]	9.573	[9.021, 9.145]	[-5597, 10179]	0.363	[0.7758, 0.8270]	[18174, 95901]
TOYOTA	395	[562.4, 569.9]	8.691	[9.096, 9.144]	[-478.3, 195.4]	0.4322	[1.022, 1.043]	[612.9, 2872.6]
SUMITOMO	350	[459.4, 465.8]	10.72	[10.10, 10.16]	[-173770, 64040]	1.053	[1.174, 1.199]	[-140110, 671280]
SOFTBANK	227	[279.0, 282.5]	23.27	[25.57, 25.79]	[-33.13, 503.5]	2.504	[4.470, 4.578]	[39.20, 1795.9]
ACOM	184	[280.8, 285.1]	37.08	[37.03, 37.24]	[-187.3, 229.2]	2.446	[4.167, 4.255]	[1003, 3364]
KINTETSU	326	[428.2, 433.3]	47.48	[58.33, 58.65]	[34.96, 35.93]	1.524	[6.573, 6.711]	[9.774, 11.72]

Table 9: Estimated parameters for the five Japanese companies

TOYOTA	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000016592	0.013558	2.6224	0.74000	0.78350
Aug-11	0.000018071	0.013069	2.8189	0.75053	0.81037
Sep-11	0.000020989	0.013076	3.0724	0.74952	0.80966
Oct-11	0.000042086	0.012979	2.5563	0.76010	0.82044
Nov-11	0.000043858	0.013200	2.7519	0.72895	0.78831
Dec-11	0.000054719	0.013196	3.1070	0.73500	0.79560
Jan-12	0.000068604	0.013220	2.2421	0.75474	0.81537
Feb-12	0.000057343	0.013059	2.6130	0.74743	0.80799
Mar-12	0.000062346	0.012976	3.7610	0.77056	0.83063
Apr-12	0.000066158	0.012493	3.5610	0.75412	0.81451
May-12	0.000073166	0.012329	3.9444	0.75400	0.81430
Jun-12	0.000066513	0.012066	2.8812	0.78222	0.84308
Jul-12	0.000072239	0.012020	3.8004	0.70337	0.75037
Aug-12	0.000072655	0.012010	3.4328	0.78632	0.84730

SUMITOMO	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000051322	0.016860	2.8330	0.68333	0.73233
Aug-11	0.000035829	0.016209	3.0178	0.67239	0.72968
Sep-11	0.000018569	0.015808	3.1157	0.65667	0.71357
Oct-11	0.000057152	0.015522	2.3621	0.65591	0.71287
Nov-11	0.000068349	0.015478	2.6475	0.65370	0.71001
Dec-11	0.000095597	0.015401	2.8006	0.66188	0.71953
Jan-12	0.000093725	0.015486	2.3190	0.67512	0.73286
Feb-12	0.000071113	0.015433	2.2408	0.67513	0.73241
Mar-12	0.000085632	0.015147	3.7455	0.69313	0.75098
Apr-12	0.000071408	0.014533	3.5322	0.65625	0.71324
May-12	0.000075297	0.014088	3.5237	0.65393	0.70860
Jun-12	0.000085411	0.013960	3.0609	0.69274	0.77090
Jul-12	0.000064635	0.013799	3.3897	0.69198	0.75398
Aug-12	0.000084683	0.013748	3.4227	0.65726	0.71513

ACOM	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.00023239	0.024453	3.1280	0.64436	0.70577
Aug-11	0.00019655	0.022699	3.0762	0.65200	0.72063
Sep-11	0.00021702	0.021843	2.6692	0.64762	0.71685
Oct-11	0.00020381	0.021404	2.3142	0.65357	0.72386
Nov-11	0.00016544	0.020913	2.2926	0.65136	0.72202
Dec-11	0.00016888	0.020536	2.3873	0.65762	0.72976
Jan-12	0.00019918	0.020533	1.7222	0.67364	0.74606
Feb-12	0.00015557	0.020452	1.6731	0.65868	0.73104
Mar-12	0.00016493	0.020007	2.7094	0.67155	0.76402
Apr-12	0.00007625	0.018931	2.3075	0.63370	0.70571
May-12	0.00007631	0.017024	2.2171	0.63196	0.70366
Jun-12	0.00011889	0.017193	2.0118	0.64653	0.71886
Jul-12	0.00013246	0.017073	2.4390	0.64387	0.72749
Aug-12	0.00013933	0.017203	2.0288	0.61761	0.68963

SOFTBANK	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.00039551	0.035201	1.6353	0.59403	0.64523
Aug-11	0.00039551	0.034311	2.0647	0.56029	0.61450
Sep-11	0.00030875	0.032377	2.0782	0.53676	0.59019
Oct-11	0.00037748	0.031750	2.0775	0.52635	0.58004
Nov-11	0.00041580	0.030709	2.7296	0.50281	0.55680
Dec-11	0.00032639	0.028730	2.7614	0.51719	0.57353
Jan-12	0.00035196	0.028725	1.9906	0.52964	0.58589
Feb-12	0.00033239	0.027831	2.2431	0.55843	0.61516
Mar-12	0.00030165	0.027340	3.4289	0.54412	0.59579
Apr-12	0.00025177	0.026916	2.9675	0.52331	0.57889
May-12	0.00029868	0.026846	3.1040	0.49694	0.55194
Jun-12	0.00030011	0.026789	2.4164	0.50513	0.56085
Jul-12	0.00023471	0.026377	2.9605	0.41663	0.46977
Aug-12	0.00024638	0.024911	2.7481	0.44226	0.49533

KINTETSU	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000059995	0.017274	3.4686	0.50163	0.55863
Aug-11	0.000055501	0.016011	3.8646	0.52618	0.59521
Sep-11	0.000062030	0.015879	3.9044	0.51444	0.58360
Oct-11	0.000107611	0.015805	3.2741	0.52735	0.59695
Nov-11	0.000131801	0.015883	2.1935	0.48865	0.54689
Dec-11	0.000143724	0.015710	3.1654	0.49152	0.55087
Jan-12	0.000141769	0.015265	2.4297	0.50676	0.56645
Feb-12	0.000130508	0.015114	3.0162	0.50758	0.56666
Mar-12	0.000133300	0.014685	4.9466	0.51266	0.57194
Apr-12	0.000126775	0.014528	4.2262	0.50576	0.56538
May-12	0.000135597	0.014079	4.9599	0.50943	0.56909
Jun-12	0.000149173	0.013938	3.1583	0.53379	0.59452
Jul-12	0.000138771	0.013638	3.9440	0.55844	0.62944
Aug-12	0.000125669	0.012694	3.8032	0.52788	0.58862

Table 10: Estimated parameters in the five U.S. companies

DELL	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000015041	0.0078406	5.6042	2.3983	2.4643
Aug-11	0.000013632	0.0076707	4.5268	2.4711	2.5540
Sep-11	0.000020935	0.0075556	5.6328	2.4554	2.5370
Oct-11	0.000020056	0.0074039	4.7260	2.4760	2.5576
Nov-11	0.000020427	0.0063918	6.3591	2.3119	2.3929
Dec-11	0.000020072	0.0063359	4.6250	2.4592	2.5417
Jan-12	0.000024623	0.0061908	3.5865	2.4671	2.5497
Feb-12	0.000020222	0.0058944	3.8172	2.5154	2.5986
Mar-12	0.000020652	0.0056130	6.1108	2.4910	2.5739
Apr-12	0.000017746	0.0053604	7.3363	2.5242	2.6063
May-12	0.000017630	0.0052944	7.3452	2.5394	2.6220
Jun-12	0.000019792	0.0052700	6.2858	2.5382	2.6200
Jul-12	0.000020123	0.0050931	5.8658	2.5438	2.6073
Aug-12	0.000019657	0.0052551	5.2596	2.5132	2.5951

AT&T	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.00022809	0.021741	3.3623	1.0487	1.1244
Aug-11	0.00020493	0.021099	2.8025	1.0774	1.1540
Sep-11	0.00020046	0.019966	2.9634	1.0554	1.1307
Oct-11	0.00019574	0.019786	2.1715	1.0913	1.1707
Nov-11	0.00019276	0.019367	2.7569	1.0915	1.1683
Dec-11	0.00018296	0.018819	2.1970	1.1378	1.2220
Jan-12	0.00018088	0.018419	1.1592	1.1406	1.2256
Feb-12	0.00016713	0.017972	2.3476	1.1795	1.2676
Mar-12	0.00015687	0.017657	3.3441	1.2119	1.3033
Apr-12	0.00016139	0.016743	3.9115	1.2019	1.2848
May-12	0.00015024	0.016092	3.6982	1.2015	1.2913
Jun-12	0.00015646	0.016051	2.2913	1.1865	1.2758
Jul-12	0.00014261	0.015947	2.5936	1.1945	1.2901
Aug-12	0.00012698	0.015991	1.6056	1.2122	1.3179

AK STEEL	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000004735	0.0054188	4.3726	4.5364	4.6039
Aug-11	0.000006972	0.0055195	2.8873	4.4333	4.5022
Sep-11	0.000016477	0.0057674	1.2767	4.3044	4.3725
Oct-11	0.000018222	0.0059747	2.6743	4.1681	4.2416
Nov-11	0.000020861	0.0061243	3.4671	4.0043	4.0775
Dec-11	0.000022800	0.0062442	2.6876	3.8319	3.9050
Jan-12	0.000023701	0.0064216	2.7144	3.7682	3.8437
Feb-12	0.000026799	0.0065889	2.2994	3.6981	3.7747
Mar-12	0.000025131	0.0069322	3.7113	3.6298	3.7098
Apr-12	0.000027452	0.0074576	3.3876	3.2857	3.3487
May-12	0.000038632	0.0082294	3.6405	2.8941	2.9560
Jun-12	0.000055330	0.0095676	3.3172	2.3093	2.3681
Jul-12	0.000060977	0.0098303	4.9799	2.1003	2.1417
Aug-12	0.000069379	0.0099172	4.9593	2.1768	2.2188

BELO	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000000038	0.0044582	5.2618	4.8533	4.9274
Aug-11	0.000000624	0.0044389	3.3532	5.1112	5.1977
Sep-11	0.000003964	0.0046206	3.8973	4.5316	4.6188
Oct-11	0.000005936	0.0048189	2.8387	4.4200	4.5091
Nov-11	0.000009107	0.0050463	3.6487	4.1507	4.2402
Dec-11	0.000008521	0.0052743	2.9813	3.9324	3.9982
Jan-12	0.000011502	0.0055705	2.8927	3.8716	3.9397
Feb-12	0.000013896	0.0058756	2.4393	3.6232	3.6926
Mar-12	0.000012508	0.0062960	4.1923	3.2246	3.2733
Apr-12	0.000018243	0.0064015	5.3993	2.6422	2.7060
May-12	0.000019070	0.0064420	5.5070	2.6430	2.7069
Jun-12	0.000019655	0.0060524	4.1221	2.6032	2.6663
Jul-12	0.000017672	0.0060318	4.1613	2.4111	2.4738
Aug-12	0.000020190	0.0059390	3.4263	2.5342	2.5713

VALERO	Brownian Subordination X_t		Hawkes process N_t		
	α	β	a	b	c
Jul-11	0.000023464	0.0091114	5.2234	3.3609	3.4746
Aug-11	0.000024063	0.0090561	4.6471	3.2800	3.4585
Sep-11	0.000033224	0.0089289	5.8908	3.2638	3.4410
Oct-11	0.000037788	0.0088179	4.3316	3.2737	3.4511
Nov-11	0.000026189	0.0077191	5.1958	3.2229	3.3998
Dec-11	0.000022476	0.0075005	4.1386	3.1940	3.4314
Jan-12	0.000018998	0.0073780	3.2156	3.2040	3.4679
Feb-12	0.000007534	0.0054632	3.9510	2.2067	2.4235
Mar-12	0.000010019	0.0050507	5.2422	2.2756	2.4932
Apr-12	0.000007558	0.0050642	6.0316	2.1982	2.4143
May-12	0.000010671	0.0051140	6.2831	2.1784	2.3939
Jun-12	0.000012933	0.0050859	8.4959	2.1794	2.4942
Jul-12	0.000011380	0.0050737	7.8070	2.2409	2.4085
Aug-12	0.000011390	0.0050759	7.7131	2.1059	2.3227

Table 11: The Second Test in TOYOTA and SUMITOMO

TOYOTA	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	300	387.9	29.31	5.417	5.617	3.69	0.3462	0.5861	69.31
Aug-11	368	418.2	13.64	7.085	5.781	-18.41	1.2230	0.6057	-50.56
Sep-11	286	454.1	58.79	8.669	7.522	-13.23	1.1751	0.8215	-30.09
Oct-11	280	356.3	27.26	11.488	10.566	-8.03	1.2686	1.0230	-19.36
Nov-11	343	395.8	15.39	9.998	12.557	25.60	0.6904	1.2986	88.09
Dec-11	273	447.7	64.01	10.151	10.118	-0.33	0.7100	1.1132	56.80
Jan-12	396	336.6	-15.00	9.552	10.089	5.63	0.4592	0.9596	108.96
Feb-12	400	381.6	-4.61	8.736	8.746	0.11	0.4952	0.8783	77.37
Mar-12	406	564.2	38.97	8.770	8.918	1.70	0.3730	1.0931	193.04
Apr-12	381	488.0	28.09	9.399	9.304	-1.00	0.3163	1.0146	220.81
May-12	408	622.9	52.68	9.114	8.999	-1.27	0.3969	1.1016	177.53
Jun-12	403	407.8	1.20	8.682	8.579	-1.19	0.3931	0.8284	110.74
Jul-12	395	564.0	42.77	8.691	9.159	5.38	0.4322	1.0330	139.02
Aug-12	389	559.1	43.72	9.379	9.448	0.74	0.4360	1.0687	145.12
Average	359.1	456.0	28.30	8.938	8.957	-0.04	0.6225	0.9589	91.91

SUMITOMO	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	300	381.0	26.99	7.160	7.661	7.01	0.2853	0.9949	248.70
Aug-11	343	440.1	28.31	8.385	7.531	-10.19	1.0644	1.003	-5.74
Sep-11	247	417.7	69.12	9.948	9.232	-7.20	1.2047	1.176	-2.41
Oct-11	259	296.0	14.27	17.634	12.419	-29.57	2.2655	1.320	-41.73
Nov-11	296	355.5	20.09	15.860	15.137	-4.56	1.5250	1.746	14.52
Dec-11	259	377.8	45.89	14.087	14.432	2.45	1.4872	1.715	15.33
Jan-12	319	318.3	-0.22	12.171	13.912	14.30	1.3930	1.5200	9.12
Feb-12	390	287.1	-26.39	10.219	10.068	-1.48	0.4094	1.027	150.85
Mar-12	370	520.5	40.68	10.083	10.185	1.01	0.2705	1.396	416.18
Apr-12	316	440.2	39.31	10.000	10.128	1.28	0.0000	1.230	
May-12	383	507.3	32.45	10.848	10.159	-6.35	1.2311	1.264	2.69
Jun-12	368	361.9	-1.65	10.694	13.894	29.92	0.8141	1.451	78.18
Jul-12	350	429.7	22.79	10.720	10.105	-5.74	1.0534	1.152	9.40
Aug-12	333	487.9	46.53	10.238	10.167	-0.69	1.1227	1.2226	8.90
Average	323.8	401.5	25.58	11.289	11.074	-0.70	1.0090	1.301	69.54

Table 12: The Second Test in SOFTBANK and KINTETSU

SOFTBANK	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	175	196.9	12.51	10.62	10.31	-2.93	0.8571	2.017	135.30
Aug-11	201	264.9	31.79	15.30	10.20	-33.31	4.156	2.288	-44.94
Sep-11	182	242.6	33.30	29.56	22.34	-24.43	10.964	4.462	-59.30
Oct-11	230	222.5	-3.27	65.26	61.93	-5.10	8.042	11.70	45.47
Nov-11	258	299.9	16.24	59.80	76.38	27.73	10.34	16.45	59.00
Dec-11	198	306.3	54.68	50.08	51.92	3.69	3.271	10.40	217.96
Jan-12	267	225.3	-15.63	45.85	48.06	4.83	6.526	8.186	25.44
Feb-12	312	246.8	-20.90	29.30	41.32	41.01	5.841	7.238	23.92
Mar-12	272	403.7	48.43	23.02	27.30	18.64	1.014	6.030	494.51
Apr-12	243	310.9	27.96	26.26	24.74	-5.77	1.769	4.709	166.27
May-12	238	334.0	40.33	34.30	31.24	-8.92	3.858	6.126	58.80
Jun-12	218	263.3	20.76	27.60	30.84	11.73	3.148	5.313	68.75
Jul-12	227	280.2	23.43	23.26	25.77	10.76	2.504	4.555	81.91
Aug-12	245	291.5	18.97	22.26	25.75	15.66	1.832	4.383	139.24
Average	233.3	277.8	20.61	33.03	34.86	3.83	4.580	6.704	100.88

KINTETSU	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	317	345.7	9.06	10.81	12.60	16.57	0.6562	1.599	143.61
Aug-11	352	434.9	23.56	13.08	11.12	-14.95	2.722	1.473	-45.87
Sep-11	283	404.1	42.77	22.28	17.69	-20.59	5.136	2.259	-56.02
Oct-11	229	321.5	40.41	50.79	35.52	-30.07	8.658	3.998	-53.83
Nov-11	274	232.0	-15.33	60.37	55.65	-7.82	5.641	5.310	-5.87
Dec-11	225	330.6	46.95	61.17	65.09	6.41	5.205	7.423	42.62
Jan-12	335	260.9	-22.11	61.09	62.92	3.00	8.749	6.177	-29.40
Feb-12	431	303.0	-29.71	40.56	47.47	17.04	3.009	4.939	64.16
Mar-12	378	531.7	40.67	47.16	41.02	-13.03	5.977	5.591	-6.47
Apr-12	391	418.0	6.91	44.53	51.09	14.73	3.794	6.056	59.61
May-12	336	566.4	68.57	54.26	47.63	-12.22	5.786	6.465	11.74
Jun-12	331	328.0	-0.91	58.52	61.15	4.49	6.328	6.157	-2.71
Jul-12	343	430.6	25.54	47.32	58.45	23.52	1.524	6.709	340.21
Aug-12	362	445.4	23.03	43.39	44.96	3.61	1.732	4.843	179.64
Average	327.6	382.3	18.53	43.95	43.74	-0.67	4.637	4.928	45.82

Table 13: The Second Test in ACOM (Japan) and DELL(U.S)

ACOM	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	282	378.2	34.13	21.246	20.688	-2.62	1.3913	3.908	180.91
Aug-11	277	412.1	48.76	32.286	23.179	-28.21	10.0987	4.252	-57.90
Sep-11	240	333.5	38.95	52.962	43.297	-18.25	10.6106	6.983	-34.19
Oct-11	200	271.4	35.70	42.682	51.191	19.94	7.3223	7.170	-2.08
Nov-11	241	286.1	18.73	29.883	40.737	36.32	6.6554	5.705	-14.28
Dec-11	173	301.5	74.29	23.651	22.069	-6.69	2.0037	3.120	55.69
Jan-12	205	221.9	8.25	21.747	22.843	5.04	1.2694	2.7348	115.44
Feb-12	234	197.3	-15.66	23.659	20.266	-14.34	2.6537	2.310	-12.95
Mar-12	219	315.0	43.82	20.455	24.535	19.95	0.8579	3.459	303.21
Apr-12	177	262.6	48.34	23.305	20.176	-13.43	1.7415	2.446	40.48
May-12	209	288.2	37.88	28.648	23.536	-17.84	6.9768	2.678	-61.62
Jun-12	209	233.1	11.53	36.597	32.838	-10.27	2.4116	3.401	41.04
Jul-12	184	287.5	56.27	37.077	37.211	0.36	2.4459	4.265	74.35
Aug-12	180	257.2	42.89	31.593	32.932	4.24	4.7664	3.5625	-25.26
Average	216.4	289.0	34.56	30.414	29.678	-1.84	4.3718	4.000	43.06

DELL	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	1206	2099.9	74.12	4.892	5.053	3.29	0.1158	0.712	514.53
Aug-11	1374	1865.3	35.76	8.607	5.015	-41.74	2.1039	0.658	-68.73
Sep-11	1426	2114.6	48.29	9.960	10.132	1.74	0.3193	1.392	335.85
Oct-11	1492	1685.8	12.99	9.928	10.086	1.60	0.1545	1.195	673.19
Nov-11	1319	2318.5	75.78	9.715	10.210	5.10	0.7162	1.260	75.96
Dec-11	1012	1760.8	73.99	9.987	10.195	2.09	0.0618	1.066	1623.32
Jan-12	1439	1365.9	-5.08	9.586	10.119	5.56	0.6165	0.9259	50.18
Feb-12	1487	1359.4	-8.58	9.607	9.093	-5.35	0.5446	0.761	39.73
Mar-12	1779	2327.4	30.83	8.874	10.165	14.55	1.0816	1.090	0.74
Apr-12	1639	2664.4	62.56	9.164	6.872	-25.01	0.9339	0.736	-21.16
May-12	1856	3060.9	64.92	10.547	8.204	-22.22	1.6904	0.966	-42.84
Jun-12	1610	2451.2	52.25	10.089	10.183	0.94	0.3967	1.069	169.44
Jul-12	1391	2545.5	83.00	12.505	10.238	-18.14	2.8544	1.069	-62.54
Aug-12	1560	2163.9	38.71	11.143	12.345	10.79	0.9461	1.2159	28.52
Average	1470.7	2127.4	45.68	9.614	9.136	-4.77	0.8954	1.008	236.87

Table 14: The Second Test in the AT&T and AK STEEL

AT&T	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	432	610.8	41.40	6.912	9.022	30.52	0.9463	1.976	108.85
Aug-11	394	545.3	38.39	6.638	6.945	4.63	0.9786	1.363	39.22
Sep-11	265	587.2	121.60	8.387	7.998	-4.64	1.0492	1.537	46.45
Oct-11	322	412.3	28.04	8.292	7.736	-6.71	0.9012	1.235	37.03
Nov-11	318	550.6	73.15	7.816	8.718	11.53	0.9176	1.579	72.03
Dec-11	188	445.9	137.18	8.181	8.869	8.41	0.5333	1.401	162.74
Jan-12	360	236.2	-34.39	7.649	8.618	12.67	0.5333	0.9373	75.74
Feb-12	440	452.0	2.72	7.762	8.780	13.11	0.5266	1.323	151.25
Mar-12	541	680.8	25.83	8.801	8.119	-7.76	0.8593	1.563	81.90
Apr-12	467	771.7	65.26	8.768	8.637	-1.49	0.8186	1.617	97.59
May-12	343	803.0	134.12	9.325	8.685	-6.86	0.8220	1.574	91.48
Jun-12	316	436.0	37.98	8.603	10.318	19.93	0.8240	1.385	68.11
Jul-12	185	518.2	180.08	7.765	8.980	15.65	0.5625	1.310	132.88
Aug-12	159	328.9	106.88	7.580	7.598	0.24	0.2649	0.8639	226.17
Average	337.9	527.1	68.45	8.034	8.502	6.37	0.7527	1.405	99.39

AK STEEL	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	714	2954.0	313.73	17.73	18.82	6.19	1.965	2.145	9.18
Aug-11	253	2215.5	775.69	46.11	20.08	-56.45	17.233	1.984	-88.48
Sep-11	578	888.4	53.70	64.15	61.30	-4.45	5.073	3.808	-24.95
Oct-11	916	1655.2	80.70	68.18	73.15	7.29	3.965	6.825	72.13
Nov-11	796	2229.2	180.05	65.70	64.63	-1.62	5.043	7.242	43.59
Dec-11	746	1628.6	118.31	65.84	66.52	1.04	3.977	6.414	61.29
Jan-12	929	1635.8	76.08	56.97	61.10	7.25	4.656	6.157	32.23
Feb-12	843	1267.5	50.36	45.86	52.67	14.83	5.761	4.735	-17.81
Mar-12	738	2082.2	182.14	48.05	41.29	-14.08	7.559	5.138	-32.03
Apr-12	747	1760.1	135.62	43.89	47.15	7.41	5.974	5.759	-3.60
May-12	767	1951.0	154.37	41.95	39.29	-6.36	5.967	5.685	-4.74
Jun-12	1579	1262.6	-20.04	50.48	46.34	-8.22	7.436	6.209	-16.51
Jul-12	1366	2046.8	49.84	55.00	40.69	-26.01	10.916	7.314	-32.99
Aug-12	1101	2255.1	104.82	49.82	67.16	34.82	9.625	12.863	33.64
Average	862.4	1845.1	161.10	51.41	50.01	-2.74	6.796	5.877	2.21

Table 15: The Second Test in BELO and VALERO

BELO	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	1192	3627.7	204.34	10.107	9.969	-1.36	0.3407	1.061	211.28
Aug-11	920	2740.5	197.88	23.315	10.095	-56.70	5.9588	0.918	-84.60
Sep-11	859	2451.3	185.37	23.818	25.039	5.13	2.5310	2.250	-11.10
Oct-11	908	1685.6	85.64	26.714	23.380	-12.48	1.8629	1.725	-7.42
Nov-11	896	2165.7	141.71	24.568	32.096	30.64	1.8964	2.914	53.65
Dec-11	808	1963.8	143.04	23.621	21.358	-9.58	2.1719	1.879	-13.47
Jan-12	1045	1800.2	72.27	21.081	23.543	11.68	1.7514	2.1072	20.32
Feb-12	1114	1315.3	18.07	19.605	18.831	-3.95	1.4117	1.573	11.43
Mar-12	1341	2437.8	81.79	18.034	19.179	6.35	1.6678	2.306	38.28
Apr-12	1237	2228.1	80.12	16.205	16.586	2.35	1.5888	1.969	23.93
May-12	1168	2658.7	127.63	15.989	15.257	-4.58	2.0106	1.971	-1.95
Jun-12	994	1713.0	72.33	15.536	16.299	4.91	1.4648	1.581	7.92
Jul-12	892	1744.2	95.54	16.739	17.695	5.71	2.0980	1.747	-16.74
Aug-12	1015	1873.2	84.55	14.630	16.561	13.19	1.2655	1.6743	32.30
Average	1027.8	2171.8	113.59	19.283	18.992	-0.62	2.0014	1.834	18.84

VALERO	Quotes			Average			Standard Deviation		
	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)	Observed	Hawkes	Error(%)
Jul-11	986	1422.9	44.31	4.992	5.089	1.95	0.03710	0.675	1719.01
Aug-11	1215	1613.3	32.78	8.973	5.130	-42.83	1.9160	0.750	-60.84
Sep-11	904	1969.4	117.85	10.085	10.129	0.43	0.5338	1.540	188.50
Oct-11	1119	1346.8	20.36	10.452	10.279	-1.65	1.2902	1.296	0.46
Nov-11	850	1683.8	98.09	9.912	9.823	-0.90	0.7199	1.208	67.85
Dec-11	604	1101.8	82.42	10.355	10.091	-2.55	1.0271	1.009	-1.72
Jan-12	797	802.3	0.67	9.800	10.116	3.22	0.2958	0.8151	175.53
Feb-12	782	755.6	-3.37	9.530	9.620	0.94	0.4218	0.561	33.04
Mar-12	890	1076.6	20.97	7.905	9.056	14.55	0.8955	0.607	-32.26
Apr-12	886	1153.8	30.23	7.935	7.001	-11.77	1.0866	0.487	-55.17
May-12	1225	1311.1	7.03	9.357	8.518	-8.96	0.8006	0.630	-21.32
Jun-12	1264	1236.3	-2.19	9.410	10.029	6.58	0.6553	0.749	14.34
Jul-12	1195	1836.6	53.69	9.573	9.090	-5.05	0.3631	0.796	119.13
Aug-12	1115	1570.2	40.83	8.761	9.041	3.19	0.9643	0.7389	-23.38
Average	988.0	1348.6	38.83	9.074	8.786	-3.06	0.7862	0.847	151.66

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